T.T.P. Series No.1

TECHNICAL MATHEMATICS

METAL TRADE

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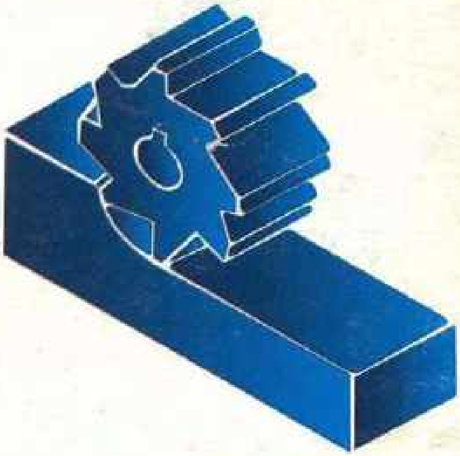

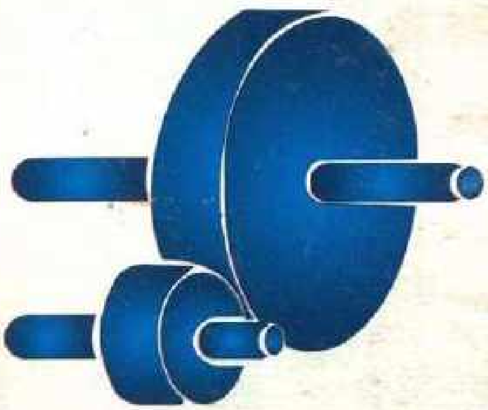

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TECHNICAL MATHEMATICS

for

METAL TRADES

Written by:

A.G. Minhas
Assistant Director,
Development Cell for
Skilled Labour Training.

Revised and improved by:

H. Thiemann
German Project Managez,
Development Cell for
Skilled Labour Training.

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FOREWORD

This textbook contains Part I & II. Part I is a revised edition of earlier publication. It is specifically designed for skilled labour training being conducted by the Directorate of Manpower & Training, Punjab.

Ever since the venture of the Technical Training and Apprentices Training Schemes began, a difficulty was felt in imparting theory instructions on account of the non-availability of suitable textbooks at this level.

In order to overcome these difficulties the
DEVELOPMENT CELL FOR SKILLED LABOUR TRAINING
has been set up under the aegis of the Directorate of Manpower and Training under the Pak-German Technical Assistance Programme and has taken in hand this task as one of the activities.

The main intention in publishing textbooks of this series is to cover the syllabi of the standardized training courses worked out under the same programme. This will help the instructors follow the courses strictly in accordance with the syllabus and help the trainees and apprentices to repeat on their own.

The use of International Units of measurement (SI-system) has been emphasized throughout the book.

It is hoped that this book will be found useful in obtaining the desired objectives. However, suggestions to make it more useful and to improve its standard would be very much appreciated and may be sent to:

The Development Cell for Skilled
Labour Training,
Ferozepur Road,

Lahore, the
5th August, 1976.

FARID UD DIN AHMAD
Director,Manpower & Training,
Punjab, LAHORE.

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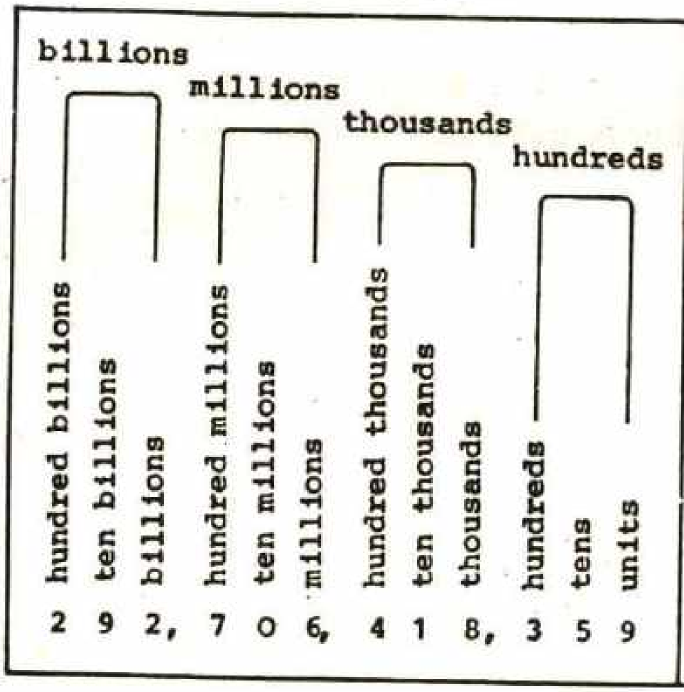
CHAPTER 1 WHOLE NUMBERS

1.1 Reading Whole Numbers



This number is read: Two hundred ninety two billion, seven hundred six million, four hundred eighteen thousand, three hundred fifty nine.

Reading a whole number or writing it in words from figure:

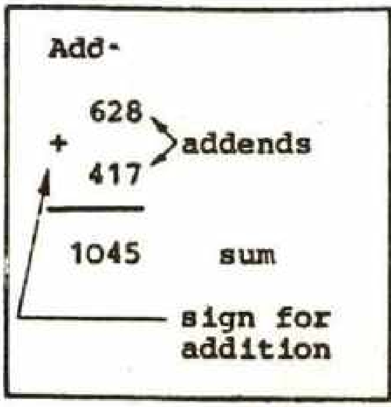
1. When the given number contains 4 or more figures, separate by commas the number into as many groups of 3 figures each as possible, starting from the unit place and going to the left.

2. Beginning from the left, read each group of figures separately, applying the name of the group indicated by each comma as it is reached.

Examples:

589,238,198	Five hundred eighty nine million, two hundred thirty eight thousand, one hundred ninety eight.
8,134,295	Eight million, one hundred thirty four thousand, two hundred ninety five.
693,281	Six hundred ninety three thousand, two hundred eighty one.
61,752	Sixty one thousand, seven hundred fifty two.
2,845	Two thousand, eight hundred forty five.
749	Seven hundred forty nine.
38	Thirty eight.
4	Four.

1.2 Addition of Whole Numbers



Addends are numbers that are added.

Sum is the answer in addition.

When adlends are interchanged, the sum does not change.

Plus (+) is the sign for addition.

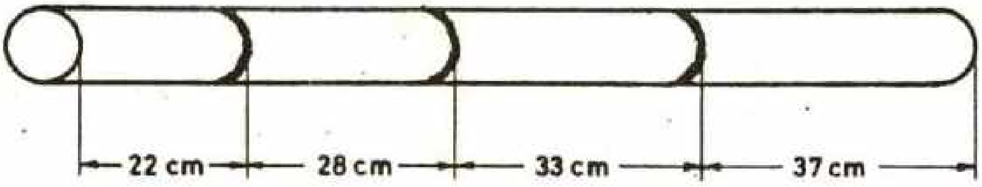
To add whole numbers:

1. Arrange the addends one under the other, with units under units, tens under tens, and so on.
2. Add each column, starting from the units column and going to the left.
3. If the sum of any column is ten or more, write the last figure under the line and carry the other figures to the top of the next column.
4. Check by adding the columns in the opposite direction.

Examples:

a) $\begin{array}{r} 52 \\ +\ 47 \\ \hline 99 \end{array}$ b) $\begin{array}{r} 356 \\ +\ 571 \\ \hline 927 \end{array}$ c) $\begin{array}{r} 2947 \\ +\ 4370 \\ \hline 7317 \end{array}$ d) $\begin{array}{r} 37372 \\ +\ 94725 \\ \hline 132097 \end{array}$

e) Calculate the total length of the pipe.



Total length = 22 cm + 28 cm + 33 cm + 37 cm

= 120 cm

$$\begin{array}{r} 22 \\ 28 \\ 33 \\ +37 \\ \hline 120 \end{array}$$

1.3 Subtraction of Whole Numbers

Subtract: 4,963 minuend - 2,597 subtrahend 2,366 difference sign for subtraction	Minuend is the number from which we subtract. Subtrahend is the number that we subtract. Difference is the answer in subtraction. Minus (-) is the sign for subtraction.

To subtract whole numbers:

1. Subtract the figures in the subtrahend from the corresponding figures in the minuend, starting from the units place and working to the left.
2. If any figure in the subtrahend is more than the corresponding figure in the minuend, increase the figure in the minuend by 10 by borrowing 1 from the preceding figure in the next higher place.
3. Check by adding the difference to the subtrahend. The sum must be equal to the minuend.

Examples:

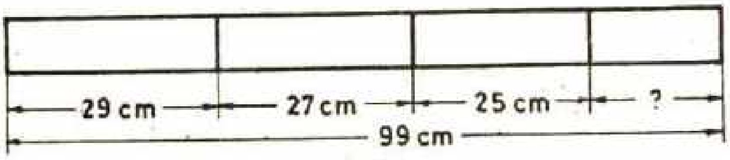
a) $$\begin{array}{r} 28 \\ -\ \underline{2} \\ 26 \end{array}$$

b) $$\begin{array}{r} 23 \\ -\ \underline{19} \\ 4 \end{array}$$

c) $$\begin{array}{r} 471 \\ -\ \underline{288} \\ 183 \end{array}$$

d) $$\begin{array}{r} 2752 \\ -\ \underline{1899} \\ 853 \end{array}$$

e)



$$\begin{array}{r} 29 \\ 27 \\ \underline{+25} \\ 81 \end{array}$$

A = 99 cm - 81 cm = 18 cm

f) There is a stock of 277 bolts available at the store. The store keeper has issued 115 bolts to Fitting section and 93 to Auto section. How many bolts are left with him?

Total stock = 277 bolts
Issued: Fitting section = 115 bolts
Auto section = 93 bolts

$$\begin{array}{r} 115 \\ \underline{+\ 93} \\ 208 \end{array}$$

Solution: Total issued = 115 + 93 bolts = 208 bolts
Left in the store = 277 - 208 bolts
= 69 bolts

$$\begin{array}{r} 277 \\ \underline{-208} \\ 69 \end{array}$$

1.4 Multiplication of Whole Numbers

Multiply:		
27	multiplicand	Multiplicand is the number that we multiply.
x 2	multiplier	Multiplier is the number by which we multiply.
54	product	Product is the answer in multiplication.
	sign for multiplication	Cross (x) is the sign for multiplication (read "times").

Multiplier containing one figure:

1. Multiply each figure in the multiplicand by the one-figured multiplier, starting from the right and working in order to the left.
2. If any product is ten or more, write the last figure of the product in the answer and add the other figure to the next product.

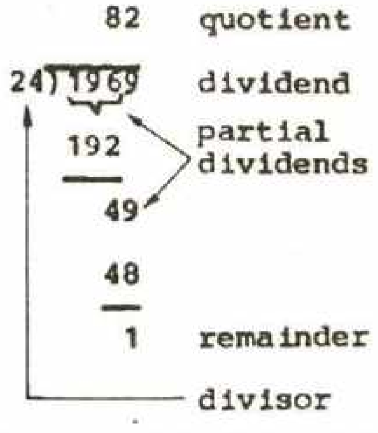
Multiply:		
48	← multiplicand	
x 57	← multiplier	Partial product is the product obtained by multiplying the multiplicand by any figure in the multiplier.
336 240	partial products	
2736	← product	

Multiplier containing two or more figures:

1. Multiply all the figures in the multiplicand by each figure in the multiplier, starting from the right and working to the left, to find the partial products.
2. Write the partial products under each other, placing the numbers so that the right-hand figure of each partial product is directly under its corresponding figure in the multiplier.
3. Add the partial products.
4. Check by interchanging the multiplier and the multiplicand and multiply again.

1.5 Division of Whole Numbers

Divide:



Divisor is the number by which we divide.

Dividend is the number that we divide.

Quotient is the answer in division.

Remainder is the number left over when the division is not exact.

Partial dividend is part of dividend annexed to remainder.

To divide whole numbers:

1. From the left, mark-off as many figures in the dividend as necessary to form a number at least larger than the divisor. This is the partial dividend.

2. Write in the space for the quotient how many times the divisor can be contained from the mark-off number. This number is the partial quotient and 9 is the largest figure that can be used in the partial quotient.

3. Multiply the whole divisor by this partial quotient, and write the product below the partial dividend. This product must be less than the partial dividend.

4. Subtract this product from the corresponding partial dividend. The difference is the remainder.

5. Bring down the next figure of the dividend and annex it to the remainder.

6. Using the remainder and the annexed figure as partial dividend repeat the process above.

7. If the last partial dividend is not divisible this is the remainder and may be written placing a R (Remainder) sign in the quotient.

8. Check by multiplying the quotient by the divisor. The product plus the remainder, if any, should equal the dividend.

Exercises:

1) Write the following numbers as word statements:

a) 79 ____________________

b) 943 ____________________

c) 3,006 ____________________

d) 719,894 ____________________

e) 75,010,254 ____________________

____

f) 18,460,050,000 ____________________

____

2) Write each of the following as a number:

a) Nine thousand, four hundred twenty six. ____________

b) Sixty nine thousand, six hundred three. ____________

c) Seven hundred five thousand, one hundred fifty. ____________

d) Three million, four hundred thousand, ninety two. ____________

e) Forty nine million, seven hundred twenty one thousand, eight hundred sixty ____________

3) Add and check:

a)	54	b)	27	c)	96	d)	35
	35		69.		87		46

e)	28	f)	23	g)	64	h)	27
	65		46		88		35
	43		79		75		82

i) 194
529
283
173

j) 668
235
879
268

k) 5,938
2,497
3,347
2,107

l) 95,769
28,645
58,987
78,329

m) 84,244
973,591
8,209
78,329
426,754

n) 25,268
2,803
45
746,482
750

o) 78,397
246,183
2,072
125,746
38

4) Subtract and check:

a) 87
3

b) 59
51

c) 842
639

d) 4,502
3,191

e) 92,846
60,524

f) 93,261
76,527

g) 720,345
471,538

h) 6,850,000
4,582,975

i) From 892,065 subtract 609,527.

j) Find the difference between 475,000 and 47,500.

5) Multiply and check:

a) 32
27

b) 74,568
95

c) 50,000
43

d) $\begin{array}{r} 8,004 \\ \underline{72} \end{array}$ e) $\begin{array}{r} 3,006 \\ \underline{2,009} \end{array}$ f) $\begin{array}{r} 90,405 \\ \underline{905} \end{array}$

g) 85 x 24 x 437 h) 27 x 273 x 876

6) Divide and check:

a) $19\overline{)\,95}$ b) $87\overline{)\,696}$ c) $26\overline{)\,884}$

d) $58\overline{)\,5,626}$ e) $79\overline{)\,38,631}$ f) $43\overline{)\,83,205}$

g) $83\overline{)\,498,581}$ h) $231\overline{)\,82,929}$ i) $765\overline{)\,716,805}$

j) $76\overline{)\,61,028}$ k) $987\overline{)\,5,922}$ l) $306\overline{)\,18,054}$

7) Problems:

a) If the average speed of an aircraft is 500 kilometres per hour, how far will it fly in 16 hours ?

b) A machine operator earns Rs 3.00 an hour. How much does he earn in a 45-hour week ?

c) If 116 kilograms of cast iron are used in the manufacture of one grinding machine, how much will be used to make 12 machines ?

d) Suppose that the cast iron in the previous question costs Rs 6.00 a kilogram. How much will be the total cost of iron for the 12 machines ?

e) A factory has 8 automatic thread rolling machines. Each can produce 178 bolts an hour. What is the total production of all the machines in 8 hours ?

f) A lathe operator earning Rs 96.- a week was appointed foreman at a salary of Rs 520.- a month. If the plant worked 50 weeks a year, by how much did his income increase ?

CHAPTER 2 FRACTIONS

2.1 Common Fractions

When a thing or a unit is divided into equal parts, one or more of the equal parts is a fraction.

Fraction:

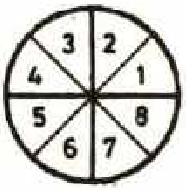
2 numerator
—
3 denominator

Numerator is the number above.

Denominator is the number below.

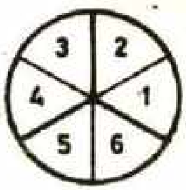
The horizontal line may be regarded as a sign of division.

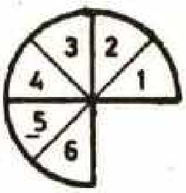
Example:



$\frac{8}{8}$ may be written as 1 whole

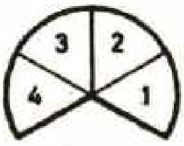
$\frac{6}{6}$ may be written as 1 whole

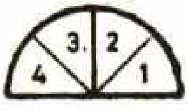




$\frac{6}{8}$ may be written as $\frac{3}{4}$

$\frac{4}{6}$ may be written as $\frac{2}{3}$





$\frac{4}{8}$ may be written as $\frac{1}{2}$

$\frac{2}{6}$ may be written as $\frac{1}{3}$

$\frac{2}{8}$ may be written as $\frac{1}{4}$

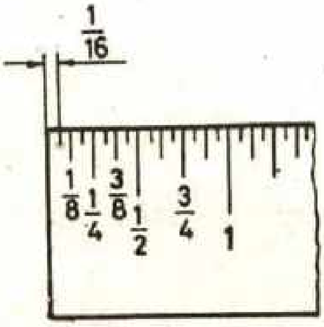
$\frac{1}{6}$

$\frac{1}{8}$

In the above examples, one circle is divided into 8 equal parts and the other into 6. Each part of the circle divided into 8 equal parts is said to be one eighth and written as 1/8. Two such parts make two eighths (2/8 or 1/4).

Each part of the circle divided into 6 equal parts is said to be one sixth and written as 1/6. Two such parts make two sixths (2/6 or 1/3).

Consider an inch, there are several number of fractions in it.



In the above illustration, an inch is divided into sixteen equal parts, each part is called "one sixteenth" written as $\frac{1}{16}$.

2.2 Addition of Fractions with like denominators

To add fractions, it is necessary to have all the denominators the same, i.e., in their lowest common denominator (L.C.D.).

Addition $\frac{1}{16} + \frac{1}{16}$	To add fractions with like denominators add the numerators and write the sum over the common denominator.
$= \frac{2}{16} = \frac{1}{8}$	Reduce answer to simplest form.

Examples:

a) $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

b) $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

c) $\frac{7}{8} + \frac{2}{8} + \frac{1}{8} + \frac{6}{8} = \frac{16}{8} = 2$

d) $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

e) $\frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$

f) $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

2.3 Addition of Fractions with different denominators

It has been established, that to add fractions having like denominators, that is, a common denominator, all we have to do is add their numerators. Thus

$$\frac{1}{16} + \frac{3}{16} + \frac{7}{16}$$

means,

1 sixteenth + 3 sixteenths + 7 sixteenths =

11 sixteenths

written as $\frac{11}{16}$

To add fractions with different denominators, as

$$\frac{1}{4} + \frac{3}{8} + \frac{1}{5}$$

we must first change the given fractions to equivalent fractions having a common denominator. For convenience, we find the Lowest Common Denominator (L.C.D.) of the given fractions.

Sometimes the L.C.D. can be obtained at sight, as in numbers 3, 9, 18. The lowest number that will evenly contain those three numbers is 18.
But in the above example, $\frac{1}{4} + \frac{3}{8} + \frac{1}{5}$ with denominators 4, 8 and 5 the L.C.D. is difficult to visualize. Solution to this problem is required.

Solution:

2	4	8	5	A
2	2	4	5	B
2	1	2	5	C
5	1	1	5	D
	1	1	1	E

The lowest common denominator (L.C.D.) of the given fractions is the continued product of the divisors 2, 2, 2, 5

L.C.D. = 2 x 2 x 2 x 5
= 40

1. Write the denominators of the given fractions as shown on left. Set of numbers in line A.

2. Select the smallest number that will divide one or more of the given set of numbers without a remainder.
The number (Divisor) is eventually 2.

3. Divide each of the given numbers by 2. Thus 2 will go into 4, two times, into 8, four times.
In cases where a number cannot be divided without a remainder, just bring down the number. The second set of numbers is now obtained, line B. Where the number one (1) is already obtained as in line C and D, just carry it down to the succeeding line.

4. Proceed as in steps 2 and 3 for the third and succeeding sets of numbers until the obtained set of numbers are all one (1) line E.

Rewriting the given fractions:

$\frac{1}{4}$	$\frac{10}{40}$	40 divided by 4 = 10 and 10 times 1 = 10, the new numerator
$\frac{3}{8}$	$\frac{15}{40}$	40 divided by 8 = 5 and 5 times 3 = 15, the new numerator
$\frac{1}{5}$	$\frac{8}{40}$	40 divided by 5 = 8 and 8 times 1 = 8, the new numerator

Adding 10 fortieths + 15 fortieths + 8 forthieths =

33 fortieths,

written as $\frac{33}{40}$.

The answer $\frac{33}{40}$ is a fraction in its <u>lowest term</u>. It is required that the answer to every problem concerning fractions must be reduced to its lowest term. A fraction is in its <u>lowest term</u> when the numerator and denominator are <u>prime</u> to each other.

Two numbers are <u>prime</u> to each other when they have no <u>common divisor</u> or <u>factor</u>.

Example a)

3 and 7

3 and 7 are prime to each other. No number can divide both of them without a remainder.

Example b)

4 and 12

4 and 12 are not prime to each other, both can be divided by 4 without a remainder.

$\frac{4 \div 4}{12 \div 4} = \frac{1}{3}$

The fraction $\frac{4}{12}$ is not in its lowest term. $\frac{1}{3}$ is in its lowest term.

PRINCIPLE: Dividing both numerator and denominator of a fraction by the same number gives a fraction equal in value to the original fraction.

Similarly

Multiplying both numerator and denominator of a fraction by the same number gives a fraction equal in value to the original fraction.

Example $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ thus, $\frac{1}{2} = \frac{3}{6}$

2.4 Subtraction of Fractions with like denominators

The rules for subtraction are the same as in addition.

Subtraction $\frac{4}{5} - \frac{3}{5}$ $= \frac{1}{5}$	To subtract fractions with like denominators, subtract the numerators and write the difference over the common denominator. Reduce answer to simplest form.

Examples:

a) $\frac{7}{8} - \frac{3}{8}$

$= \frac{4}{8} = \frac{1}{2}$

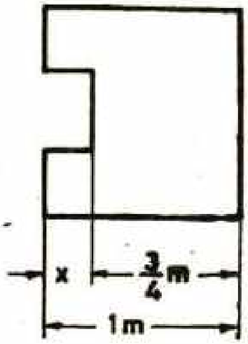
b) $\frac{11}{16} - \frac{7}{16}$

$= \frac{4}{16} = \frac{1}{4}$

c) $\frac{5}{6} - \frac{2}{6}$

$= \frac{3}{6} = \frac{1}{2}$

d) Calculate the distance "x" in the figure below.



$x = 1\text{ m} - \frac{3}{4}\text{ m}$

1 may be written as $\frac{4}{4}$

$x = \frac{4}{4}\text{ m} - \frac{3}{4}\text{ m}$

$x = \frac{1}{4}\text{ m.}$

2.5 Subtraction of Fractions with different denominators

The subtraction of fractions with different denominators is carried out in the same way as in addition. The fractions must be changed first to equivalent fractions with a common denominator.

Example a) Take $\frac{2}{3}$ from $\frac{7}{8}$

Solution:

$$\begin{array}{c|c} \frac{7}{8} & \frac{21}{24} \\ & \\ \frac{2}{3} & \frac{16}{24} \\ \hline & \frac{5}{24} \end{array}$$

The L.C.D. of the given fractions is 24, hence the equivalent fractions are $\frac{21}{24}$ and $\frac{16}{24}$.

Subtracting:

21 twenty-fourths minus 16 twenty-fourths =
5 twenty-fourths, written as $\frac{5}{24}$.

Example b) Subtract $3\frac{7}{8}$ from $4\frac{1}{2}$.

Solution:

$$\begin{array}{c|c} 4\frac{1}{2} & \frac{9}{2} = \frac{36}{8} \\ & \\ 3\frac{7}{8} & \frac{31}{8} = \frac{31}{8} \\ \hline & \qquad \frac{5}{8} \end{array}$$

Change mixed numbers to improper fractions.
Reduce to equivalent fractions with a least common denominator.

Subtracting:

36 eighths minus 31 eighths = 5 eighths, written as $\frac{5}{8}$.

2.6 Proper Fraction, Improper Fraction and Mixed Numbers

The Proper Fraction

When the numerator is smaller than the denominator, such fractions are called proper fractions.

Examples: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{11}$

The fraction is in its lowest term, when the numerator and denominator are prime to each other. The numbers are prime to each other, when they have no common factor or divisor.

Example a) $\frac{3}{5}$ 3 and 5 are prime to each other. No number can divide both of them without remainder.

While solving problems concerning fractions, the answer must be reduced to its lowest term.
As in the example $\frac{3}{5}$ cannot be further simplified, since both the numbers 3 and 5 have no common factor or divisor.
Therefore, the fraction $\frac{3}{5}$ is in its lowest term.

Example b) $\frac{4}{8}$ 4 and 8 are not prime to each other. Both have a common divisor or factor, i.e. 4.
The fraction is therefore not in its lowest term.

$\frac{1}{2}$ is in its lowest term.

Example c) Convert the following fractions to their lowest term.
i) $\frac{9}{27}$; ii) $\frac{8}{36}$; iii) $\frac{27}{55}$.

i) $\frac{9}{27}$ 9 is the common divisor to numerator and denominator.

$\therefore \frac{1}{3}$ is the lowest term of the fraction.

ii) $\frac{8}{36}$ 4 is the common divisor.

$\therefore \frac{2}{9}$ is the lowest term of the fraction.

iii) $\frac{27}{55}$ The numerator and denominator are prime to each other, therefore the fraction is in its lowest term.

The Improper Fraction

There are instances when the numerator is greater than, or equal to the denominator, such fraction is called improper fraction.

Examples: $\frac{9}{7}$, $\frac{8}{3}$, $\frac{8}{8}$, $\frac{4}{4}$

Inasmuch as the horizontal line between the numerator and denominator is an indicated division, to simplify improper fraction is to perform the indicated division.

Example a)

$\frac{9}{7} = 1\frac{2}{7}$

One unit consists of 7 sevenths, or $\frac{7}{7}$, hence in 9 sevenths there are still 2 sevenths that remain, so it is 1 and 2 sevenths,

written as $1\frac{2}{7}$.

Example b)

$\frac{29}{9} = 3\frac{2}{9}$

In this example one unit consists of 9 ninths, 3 units consist of 27 ninths,

or $\frac{27}{9} = 3$.

In 29 ninths there are still 2 ninths that remain, so the fraction can be written as

$3\frac{2}{9}$.

Such numbers as written in examples a) and b), i.e.

$1\frac{2}{7}$ and $3\frac{2}{9}$

are called mixed numbers.

They consist of a whole number and a fraction.

Mixed Numbers

Mixed number, as has been said, is composed of a whole number and a fraction. It is evident, that the sum of mixed numbers is equal to the sum of the whole numbers added to the sum of the fractions.

Example a) Add $1\frac{1}{8}$, $2\frac{7}{16}$, $3\frac{1}{4}$.

Solution:

Whole Numbers	Fractions	Equivalent Fraction with L.C.D.
1	$\frac{1}{8}$	$\frac{2}{16}$
2	$\frac{7}{16}$	$\frac{7}{16}$
3	$\frac{1}{4}$	$\frac{4}{16}$
6		$\frac{13}{16}$

$\frac{13}{16}$ is found to be the sum of the fractions, added to the sum of the whole numbers, 6, gives 6 and 13 sixteenths, written as $6\frac{13}{16}$.

Example b) Add $2\frac{3}{4}$, $3\frac{7}{8}$.

Solution:

$2\frac{3}{4} = \frac{11}{4}$

Change mixed numbers to improper fractions. Since every unit contains 4 fourths, two units contain 2 times 4 fourths, or 8 fourths, added to 3 fourths gives 11 fourths, written as $\frac{11}{4}$.

$3\frac{7}{8} = \frac{31}{8}$

Three units contain 3 times 8 eighths, or 24 eighths, added to 7 eighths gives 31 eighths, written as $\frac{31}{8}$.

Rewriting,

Improper Fraction	Equivalent Fraction
$\frac{11}{4}$	$\frac{22}{8}$
$\frac{31}{8}$	$\frac{31}{8}$

22 eighths plus 31 eighths = 53 eighths written as $\frac{53}{8}$ simplifying, $6\frac{5}{8}$.

The procedure given in Example b) is another way of solving problems involving mixed numbers.

2.7 Multiplication of Fractions

Multiply:	Multiply the numerators and write the product above the line.
${}_{3}\frac{2}{\cancel{9}} \times \frac{\cancel{3}^{1}}{5}$	Multiply the denominators and write the product below the line.
$= \frac{2}{3} \times \frac{1}{5}$	Where possible, first divide any numerator and denominator by the largest possible number that is exactly contained in both. This process is called cancellation.
$= \frac{2}{15}$	Reduce product to simplest form.

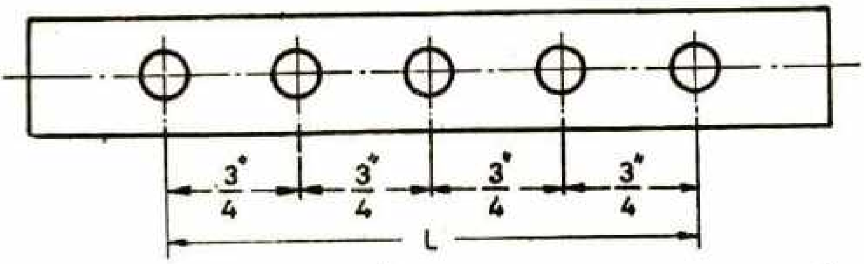
Examples:

a) $\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$

b) ${}_{4}\frac{1}{\cancel{8}} \times \frac{\cancel{2}^{1}}{7} = \frac{1}{4} \times \frac{1}{7} = \frac{1}{28}$

c) Five holes are to be drilled at $\frac{3}{4}''$ pitch.

What is the centre distance between the two end holes?



$L = \frac{3}{4}'' \times 4$

To avoid confusion 1 is placed as the denominator of 4. It does not change the value of 4.

$= \frac{3}{\cancel{4}}'' \times \frac{\cancel{4}}{1}$

$= \frac{3}{1}'' = 3''$

This example is useful in marking out, only when the divider point coincides with the last centre dot, the setting is correct and the work can now be scribed and centre punched.

d) Multiply $\frac{2}{3}$ by $\frac{9}{16}$

$$\frac{{}^{1}\cancel{2}}{{}_{1}\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{16}_{8}} = \frac{3}{8}$$

The method shown is very convenient and requires a shorter time to perform. This method is called <u>cancellation</u>.

Cancellation consists of striking out the factors that are common to both numerators and denominators of the given fraction.
3 is common to 3 and 9, while 2 is common to 2 and 16.

e) Multiply $2\frac{1}{2}$, $3\frac{2}{3}$, $1\frac{1}{5}$, use cancellation.

$$2\frac{1}{2} \times 3\frac{2}{3} \times 1\frac{1}{5} =$$

$$\frac{{}^{1}\cancel{5}}{\cancel{2}_{1}} \times \frac{11}{\cancel{3}_{1}} \times \frac{\cancel{6}^{\cancel{2}^{1}}}{\cancel{5}_{1}} =$$

$$\frac{11}{1} \text{ or } 11$$

/ Reduce to improper fractions.

/ Cancel numbers with the same factors in the numerator to that of the denominators.

/ The product of the remaining numerators 1, 11 and 1 is the numerator of the answer. Similarly, the product of the remaining denominators 1, 1 and 1 gives the denominator of the answer.

2.8 Division of Fractions

Divide:

$$\frac{3}{16} \div \frac{9}{32}$$

Invert the divisor (number after the the division sign) then replace the division sign by a multiplication sign.

$$= \frac{\cancel{3}^{1}}{\cancel{16}_{1}} \times \frac{\cancel{32}^{2}}{\cancel{9}_{3}}$$

Multiply as in multiplication of fractions, using cancellation where possible.

$$= \frac{1}{1} \times \frac{2}{3}$$

Reduce the fraction to simplest form.

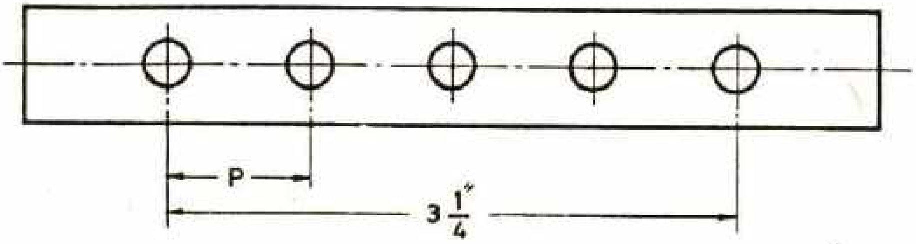
$$= \frac{2}{3}$$

Examples:

a) $\frac{1}{3} \div \frac{3}{5} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}$

b) $\frac{3}{4} \div \frac{15}{16} = \frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{\cancel{15}_{5}} = \frac{4}{5}$

c) Five holes equally spaced are to be drilled in the strip shown below. What is the pitch, P ?



$$P = \frac{3\frac{1}{4}''}{4} = 3\frac{1}{4}'' \div \frac{4}{1}$$

$$P = \frac{13}{4}'' \times \frac{1}{4}$$

$$P = \frac{13}{16}''$$

d) Divide $\frac{3}{4}$ by $\frac{2}{3}$

$$\frac{3}{4} \div \frac{2}{3} =$$

Invert the divisor $\frac{2}{3}$, giving $\frac{3}{2}$, then proceed to multiplication of fractions.

$$\frac{3}{4} \times \frac{3}{2} = \frac{9}{8} \text{ or } 1\frac{1}{8}$$

e) Divide $\frac{7}{8}$ by 2

$$\frac{7}{8} \div 2 = \frac{7}{8} \div \frac{2}{1} =$$

It is understood, that whenever a whole number appears, the denominator is 1.
Proceed as in example d).

$$\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$$

f) Divide $2\frac{1}{3}$ by $1\frac{3}{8}$

$$2\frac{1}{3} \div 1\frac{3}{8} = \frac{7}{3} \div \frac{11}{8} =$$

Change mixed numbers to improper fractions. Invert divisor and proceed to multiplication of fractions.

$$\frac{7}{3} \times \frac{8}{11} = \frac{56}{33} \text{ or } 1\frac{23}{33}$$

Problems involving Multiplication and Division

Example. Evaluate $2\frac{1}{2} \times 3\frac{3}{4} \div 1\frac{2}{3}$

Solution:

$$2\frac{1}{2} \times 3\frac{3}{4} \div 1\frac{2}{3} =$$

$$\frac{5}{2} \times \frac{15}{4} \div \frac{5}{3} =$$

$$\frac{\cancel{5}}{2} \times \frac{15}{4} \times \frac{3}{\cancel{5}} = \frac{45}{8} \text{ or } 5\frac{5}{8}$$

Reduce given fractions to improper fractions. Invert the divisor, $\frac{5}{3}$ giving $\frac{3}{5}$. Proceed to multiplication of fractions.

Complex Fractions

Example. Evaluate $\left(2\frac{1}{2} + 1\frac{3}{4}\right) \div \left(2\frac{1}{6} - 1\frac{2}{3}\right)$

An expression like this is called complex fraction. The numerator or the denominator or both are fractions or mixed numbers.

Solution:

$$\frac{2\frac{1}{2} + 1\frac{3}{4}}{2\frac{1}{6} - 1\frac{2}{3}}$$

$$= \frac{\frac{5}{2} + \frac{7}{4}}{\frac{13}{6} - \frac{5}{3}}$$

$$= \frac{\frac{10}{4} + \frac{7}{4}}{\frac{13}{6} - \frac{10}{6}} = \frac{\frac{17}{4}}{\frac{3}{6}}$$

$$= \frac{17}{4} \div \frac{3}{6} = \frac{17}{\cancel{4}_2} \times \frac{\cancel{6}^{\cancel{3}^1}}{\cancel{3}_1}$$

$$= \frac{17}{2} = 8\frac{1}{2}$$

1. Simplify first the numerator and denominator by performing the indicated addition and subtraction respectively:
   A. Change to improper fractions.
   B. Get the L.C.D. of numerator and add, giving $\frac{17}{4}$.
   C. Get the L.C.D. of denominator and subtract, giving $\frac{3}{6}$.
2. Perform the indicated division. Invert the divisor $\frac{3}{6}$, and proceed to multiplication of fractions.
3. Simplify improper fraction $\frac{17}{2}$, giving mixed number $8\frac{1}{2}$.

Exercises:

Solve the problems and reduce the answer to its lowest term.

1) $\frac{1}{8} + \frac{5}{8} + \frac{7}{8} + \frac{3}{8}$

2) $\frac{11}{16} - \frac{15}{16} + \frac{8}{16} + \frac{1}{16}$

3) $\frac{16}{3} - \frac{2}{3} - \frac{7}{3} - \frac{4}{3} - \frac{1}{3}$

4) $\frac{7}{12} + \frac{3}{10} + \frac{8}{15}$

5) $\frac{2}{5} + \frac{5}{9} + \frac{1}{8} + \frac{7}{20}$

6) $\frac{15}{16} + \frac{8}{3} + \frac{2}{3}$

7) $\frac{8}{5} - \frac{3}{7} - \frac{3}{8} - \frac{3}{10}$

8) $\frac{9}{8} - \frac{2}{3} - \frac{2}{7} - \frac{3}{5}$

9) $\frac{9}{16} - \frac{7}{8} + \frac{15}{32}$

10) $\frac{19}{28} + \frac{32}{35} - \frac{16}{21} - \frac{21}{49}$

11) $\frac{7}{9} + \frac{3}{5} + \frac{9}{16} + \frac{11}{15}$

12) $2\frac{1}{2} + 3\frac{1}{2} + 4\frac{1}{2} + 5\frac{1}{2}$

13) $5\frac{3}{4} - 6\frac{1}{2} - 4\frac{3}{8} + 5\frac{7}{8}$

14) $2\frac{1}{5} + 4\frac{1}{4} - 8\frac{11}{12} + 5\frac{5}{9}$

15) $6\frac{7}{16} + \frac{7}{8} + \frac{2}{3} + 10\frac{15}{16}$

16) $4 + 11\frac{3}{5} - 4\frac{5}{6} - 3\frac{2}{3} - 2\frac{1}{6}$

17) $14\frac{3}{10} - 8\frac{7}{15} - 6\frac{9}{20} + 2\frac{17}{30}$

18) $5\frac{2}{3} + \frac{49}{240} + 4\frac{7}{120} + 8\frac{21}{80}$

19) $\frac{15}{64} - \frac{13}{32} - \frac{5}{8} + \frac{53}{64}$

20) $\frac{19}{32} + \frac{11}{16} + 4\frac{5}{6} + 8\frac{1}{2}$

21) $1\frac{3}{4} - 16 + 8\frac{1}{12} + 9\frac{13}{16}$

22) $5 - 8\frac{1}{2} - 3\frac{3}{4} + 7\frac{1}{12}$

23) $\frac{1}{2} \times 8$

24) $\frac{5}{8} \times 7$

25) $\frac{1}{2}$ of $\frac{3}{4}$

26) $\frac{2}{3}$ of $1\frac{1}{2}$

27) $\frac{3}{4} \times \frac{4}{9} \times 3\frac{1}{4}$

28) $2\frac{3}{4} \times 5$

Hint: The word "of" has exactly the same meaning as the multiplication sign (x)

29) Problems:

a) To find the circumference of the circle, we multiply the diameter by $\frac{22}{7}$. Find the circumference of a circle whose diameter is $6\frac{1}{2}$ cm.

b) 20 blocks each $2\frac{1}{2}$ cm long are to be piled up. What is the height of the pile ?

c) To drill one hole of 10 mm dia takes $15\frac{1}{2}$ seconds. How much time is required to drill 17 holes ?

d) If the weight of the drilled out metal from the hole is 135 g in question no. c), what is the total weight of the drilled out material from 17 holes ?

30) $\frac{5}{8} \div \frac{10}{11}$

31) $\frac{1}{3} \div \frac{5}{3}$

32) $4\frac{3}{8} \div 6$

33) $4\frac{1}{3} \div 5\frac{1}{3}$

34) Problems:

a) In a mild steel stripe, 110 cm long and 20 cm wide, 15 holes are required to be drilled at equal space. What would be the distance from centre to centre of adjacent holes.

b) In a mild steel plate 20 holes were drilled. The total time taken was 20 min 25 sec. What is the time of drilling one hole ?

CHAPTER 3 DECIMAL FRACTIONS

3.1 The Decimal Fraction

A decimal fraction is a fraction whose denominator is 10 or some power of 10 as 100, 1000, 10 000 and so on. Thus

$\frac{1}{10}, \frac{25}{100}, \frac{125}{1000}$ are decimal fractions.

$$\frac{1}{10} = 0.1$$

$$\frac{25}{100} = 0.25$$

$$\frac{125}{1000} = 0.125$$

It can be observed, that the numerator of the fraction is the number after the decimal point with as many numbers of figures as there are zeros or ciphers in the denominator.

The relative values of the figures in any number is clearly illustrated by the following table.

100. = One hundred

10. = Ten

1. = One

0.1 = One tenth

0.01 = One hundredth

0.001 = One thousandth

0.0001 = One ten-thousandth

0.00001 = One hundred-thousandth

Examples:

a) 0.047 = forty seven thousandths

b) 0.1001 = one thousand one ten-thousandths

c) 3.3125 = three and three thousand one hundred twenty five ten-thousandths

d) 0.000635 = six hundred thirty five millionths

e) When we say that the accuracy of a vernier caliper is one tenth of a millimeter, it means

$$\frac{1}{10} \text{ mm or } 0.1 \text{ mm.}$$

f) When we say that the accuracy of a micrometer is one hundredth of a millimeter, it means

$$\frac{1}{100} \text{ mm or } 0.01 \text{ mm.}$$

3.2 To Reduce a Decimal Fraction to a Common Fraction

Example a) Reduce 0.005 to common fraction.

Solution:

$$0.005 = \frac{5}{1000}$$

The number after the decimal point is the numerator of the fraction, while the denominator is 1 with three zeros or ciphers (1000), because there are three figures to the right of the decimal point. The fraction obtained is still to be reduced to its lowest term.

$$\frac{{}^{1}\cancel{5}}{\underset{200}{\cancel{1000}}} = \frac{1}{200}$$

Cancelling out; 5 will go into 5, one time; into 1000, two hundred times.

$$\therefore\ 0.005 = \frac{1}{200}$$

$\frac{1}{200}$ is in its lowest term.

Example b) Reduce 0.96875 to common fraction.

Solution:

$$0.96875 = \frac{96875}{100000}$$

96875 is the numerator and the denominator is 1 with five zeros. The factor of the divisor of the two numbers cannot be obtained at sight. Select a trial divisor that will divide the two numbers without a remainder.
Try 125. It will go into 96875 seven hundred seventy five times, into 100,000 eight hundred times.

$$= \frac{96875 \div 125}{100000 \div 125}$$

$$= \frac{775 \div 25}{800 \div 25}$$

$\frac{775}{800}$ is still not in its lowest term.

$$= \frac{31}{32}$$

Further divide, until obtaining numbers that are prime to each other.

$$\therefore\ 0.96875 = \frac{31}{32}$$

$\frac{31}{32}$ is in its lowest term.

3.3 Decadic multiples or parts of a unit

multiple/part	designation	symbol	unit metre	unit gram	unit litre
1 000 000	mega	M	megametre	megagram	megalitre
1 000	kilo	k	kilometre	kilogram	kilolitre
100	hecto	h	hectometre	hectogram	hectolitre
10	deca	da	decametre	decagram	decalitre
1	--	-	metre	gram	litre
0.1	deci	d	decimetre	decigram	decilitre
0.01	centi	c	centimetre	centigram	centilitre
0.001	milli	m	millimetre	milligram	millilitre
0.000 001	micro	μ	micrometre	microgram	microlitre

Note: The symbol of the multiple/part of a unit is always written in front of the symbol of the unit.

Examples: 1 centilitre = 1 cm; 1 kilogram = 1 kg
1 millilitre = 1 ml

Change of multiples/parts of a unit

There are instances when a unit given in one multiple or part has to be changed into another one.

Change: 1 cm into m Write: 1 cm = 0.01 m
2 cm into m 2 cm = 0.01 m x 2 = 0.02 m
10 cm into m 10 cm = 0.01 m x 10 = 0.10 m
35 cm into m 35 cm = 0.01 m x 35 = 0.35 m
1 mm into m 1 mm = 0.001 m
2 mm into m 2 mm = 0.001 m x 2 = 0.002 m
35 mm into m 35 mm = 0.001 m x 35 = 0.035 m
1 km into m 1 km = 1000 m
2 km into m 2 km = 1000 m x 2 = 2000 m
10 km into m 10 km = 1000 m x 10 = 10 000 m
35 km into m 35 km = 1000 m x 35 = 35 000 m

When there is a need of changing from a multiple to a part of a unit, proceed as follows:

Change 8 cm into km: Count the number of steps cm-dm-m-dam-hm-km.
Divide for each step by 10

$$\frac{8}{10 \times 10 \times 10 \times 10 \times 10} = \frac{8}{100\ 000} = 0.00008 \text{ km}$$

3.4 Addition of Decimals

Example: Add 1.728, 0.0084, 6.52

1.728
0.0084 → addends
6.52
8.2564

Write given numbers with the decimal points in the same column.
Add as in ordinary whole numbers.
The decimal point of the sum is in the same column as the other decimal points.

3.5 Subtraction of Decimals

Example: From 125.68 take 82.875

125.680 ← minuend
82.875 ← subtrahend
42.805 ← difference

Write given numbers with the decimal point in the same column.
If the minuend contains fewer figures after the decimal point annex zeros.
Proceed as in subtraction of ordinary whole numbers.
The decimal point of the difference is in the same column as the other decimal points.

3.6 Multiplication of Decimals

Example: Multiply 27.63 by 8.322

27.63 Multiplicand
8.322 Multiplier
5526
5526
8289
22104
229.93686 Product

Multiply as in ordinary whole numbers.
Beginning from the right, count off in the product as many decimal places as there are decimal places in both multiplicand and multiplier.

In the above example, the multiplicand contains 2 decimal places, counting from right to the decimal point, and the multiplier contains 3 decimal places. Hence, the product must have 5 decimal figures counting from the right.

The decimal part of the product 229.93686 denotes hundred-thousandths. It is sometimes impractical for a craftsman to use such small fractions of a unit. The degree of accuracy for such work must be decided upon. For this particular example, we decide on rewriting it to the nearest thousandths, a process known as "rounding off figures".

Rounding off to the nearest thousandths:

229.93686 = 229.937

The figures after the third decimal place were dropped off but the third figure 6, became 7.

3.7 Rounding off Figures

Example a) State 2.94276 to three decimal places.

Solution:

```
   ┌Last figure to
   │be retained
   ↓
2.94276  =  2.943
    ↑
    └First figure to
     be dropped off
```

Three figures are to be retained after the decimal, therefore the remaining figures are to be dropped off. If the first figure to be dropped off is greater than 5, add 1 to the third figure, i.e. last figure to be retained.

Example b) Write 7.5846 to the nearest hundredths.

Solution:

```
  ┌Last figure to
  │be retained
  ↓
7.5846  =  7.58
   ↑
   └First figure to
    be dropped off
```

Two figures after the decimal are to be retained. If the first figure to be dropped off is less than 5, the last figure to be retained remains unchanged.

Example c) State 2.6365 and 2.6375 to three decimal places.

Solution:

```
         ┌Last figure to
         │be retained
         ↓
A. 2.6365  =  2.636
          ↑
          └First figure to
           be dropped off

B. 2.6375  =  2.638
```

If the first figure to be dropped off is 5, the last figure to be retained remains unchanged if it is even, as in A., and add 1 if it is odd, as in B.

3.8 Division of Decimals

Example a) Divide 24.72 by 12.

Solution:

```
Divisor       2.06   Quotient
   ──→12   )24.72   Dividend
            24
            ───
              72
              72
            ─────
```

Divide as in whole numbers and place the decimal point of the quotient above the decimal point of the dividend.

Example b) Divide 0.74 by 0.873. State the answer to three significant figures.

Solution:

$$
\begin{array}{lr}
 & 0.8476 \\
0.873 \;\big) & 0.740\,0000 \\
 & \underline{6984} \\
 & 4160 \\
 & \underline{3492} \\
 & 6680 \\
 & \underline{6111} \\
 & 5690 \\
 & \underline{5238} \\
 & 452
\end{array}
$$

Convert the divisor into a whole number by moving the decimal point 3 places to the right.
Move the decimal point of the dividend an equal number of places to the right as in the divisor.
Annex zeros.
Proceed to divide as in example a).

$$\therefore \quad \frac{0.74}{0.873} = 0.848$$

Inasmuch as the quotient must only be correct to three significant figures, we stop dividing after four significant figures for the basis of rounding off the quotient.

Rounding off to three significant figures, 6 is greater than 5, therefore, we add 1 to 7.
Thus the quotient is 0.848.

3.9 Conversion of inch into metric system

In computation it is often necessary to convert the dimensions from one unit into another. Before performing the required operation, all the dimensions should be in the same unit and system of measurement. In the British system of measurement the linear units generally used are inch, foot, yard.

1 yard (yd) = 3 feet

1 foot (ft) = 12 inches (in)

But nowadays the international units of linear measurement, i.e. millimetre, centimetre and metre, are used in Pakistan. Therfore conversion of units from the British system into metric system is important.

Conversion table from inch into metric system

1 in = 25.4 mm = 2.54 cm

1 ft = 30.48 cm

1 yd = 0.9144 m

Conversion table from metric into inch system

1 mm = 0.03937 in

1 cm = 0.3937 in

1 m = 3.28 ft = 1.0936 yd

Exercises:

1) Read the following numbers:

    a) 0.3

    b) 0.1001

    c) 0.001

    d) 0.049

    e) 0.0000635

    f) 5.6632

    g) 2.763

    h) 1.1002

    i) 3.0102

2) Read the following numbers and write in decimals:

    a) Two hundredths ....

    b) Five thousandths ....

    c) One hundred twenty-five ten-thousandths ....

    d) Five and one thousand one ten-thousandths ....

    e) Two and six hundred twenty-five ten-thousandths ....

3) Perform the indicated operations:

    a) 1284.008 m + 68.63 m

    b) 0.0008 cm + 2.08 m

    c) 0.732 m + 4.896 cm + 0.153 m + 2.404 cm + 0.889 m

    d) 100.001 cm + 24.06 cm + 85.234 m + 23.4501 cm

    e) 264 kg + 0.023 kg +0.025 kg+ 200 g +0.001 kg+ 206 g

    f) From 218.007 kg take 148.35 kg

    g) From 5.9203 m take 2.79 m

    h) Take 0.4058 m from 0.4948 m

    i) From 2.31 kg take 0.0036 kg

    j) Take 0.992 m from 2.01 m

    k) 11.52 m - 75 cm

    l) 5.09 m - 118 cm

    m) 128.92 kg - 759 g

    n) 3.1416 x 2.7 x 2.8 m

    o) 0.7854 x 3.25 m x 2.25

    p) 5280 m x 0.875

    q) 0.0003 kg x 0.025 x 1.02

    r) 231 x 2.576 x 0.0001 x 100 kg

s) Rewrite the answers to exercises n - r to three decimal places.

t) 124.625 m ÷ 48

u) 81.63 m ÷ 0.9

v) 7 kg ÷ 1.728

w) 6.455 kg ÷ 0.08

x) 0.0406 m ÷ 40.08

y) 1 m ÷ 2.375

4) Convert the following inch dimensions into mm and cm:

a) $7\frac{3}{4}''$

b) $6\frac{1}{2}''$

c) $2\frac{1}{4}''$

d) $\frac{3}{4}''$

e) $6\frac{9}{16}''$

f) $6\frac{9}{32}''$

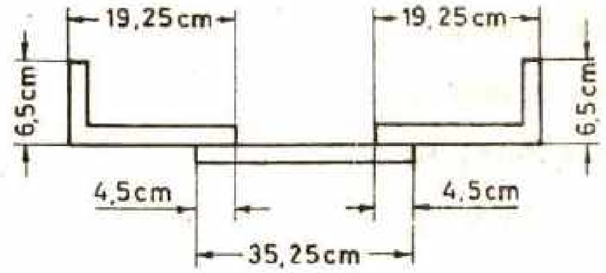
g) $5\frac{11}{16}''$

h) $9\frac{5}{8}''$

i) $3\frac{7}{8}''$

5) Problems:

a) Find the total length of the joined steel bars.



b) The actual inside diameter of a pipe is $1\frac{1}{8}$ inches, while the actual outside diameter is $1\frac{5}{16}$ inches. Find the thickness of the pipe.

c) Find the difference in diameter of the piece of tapered work in Fig. 1.

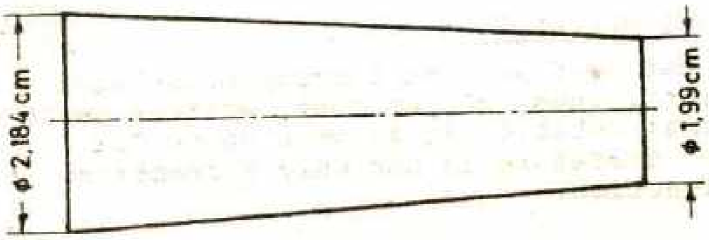


Fig. 1

d) The circumference of a circle is found by multiplying its diameter by 3.14. Find the circumference of the circle in Fig. 2. State answers to two decimal places.

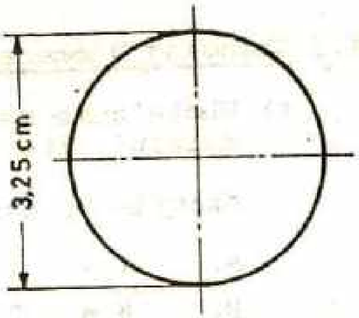


Fig. 2

e) If the circumference of the circle in Fig. 2 is 7.94 cm, find the diameter. State answer to two decimal places.

f) Find the diameter at A of the taper drill shank in Fig. 3, if the difference between the small diameter and the diameter at A is 0.032 cm.

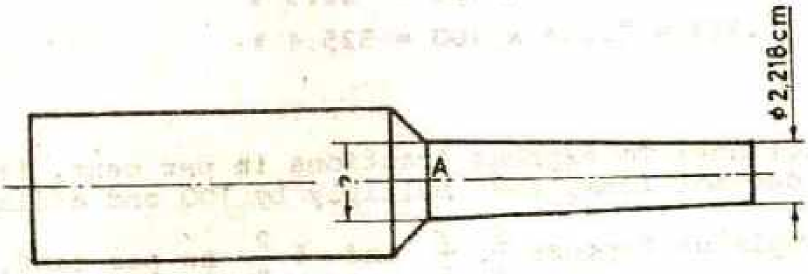


Fig. 3

g) The cutting speed on cylindrical work is found by multiplying the circumference of the work by the number of revolutions per minute. Find the cutting speed on a piece of work whose circumference is 7.964 cm. The work piece is making 120 revolutions per minute. Express the answer in metre per minute.

CHAPTER 4 PERCENTAGES

4.1 The Meaning of Per Cent

The term per cent and its corresponding sign % means "hundredths". Thus, 20 per cent, written as 20 %, means $\frac{20}{100}$. In a decimal notation it is written as 0.2.
A per cent therefore is not only a fraction; it is a decimal fraction.

4.2 Changing Numbers to Per Cent

a) Whole numbers: To change whole numbers to per cent, multiply the numbers by 100 and affix the sign %.

Example a) Express whole numbers 1, 8 and 125 as per cent.

A. $1 = 1 \times 100 = 100$ %

B. $8 = 8 \times 100 = 800$ %

C $125 = 125 \times 100 = 12500$ %

b) Decimal fractions: To change decimal fractions to per cent, also multiply the decimal fractions by 100 and affix the sign %.

Example b) Express 0.325, 0.365 and 5.254 as per cent.

A. $0.325 = 0.325 \times 100 = 32.5$ %

B. $0.365 = 0.365 \times 100 = 36.5$ %

C. $5.254 = 5.254 \times 100 = 525.4$ %

c) Fractions: To express fractions in per cent, first change to decimal form, then multiply by 100 and affix the sign %.

Example c) Express $\frac{3}{4}$, $\frac{3}{8}$ and $5\frac{2}{3}$ as per cent.

A. $\frac{3}{4} = 0.75, \quad 0.75 \times 100 = 75$ %

B. $\frac{3}{8} = 0.375, \quad 0.375 \times 100 = 37.5$ %

C. $5\frac{2}{3} = 5.66, \quad 5.66 \times 100 = 566$ %

4.3 Changing Per Cents to Decimals

To change per cent to decimal, drop the sign (%) and divide by 100.

Examples: Change $\frac{1}{2}$ %, 6.6 %, 0.003 % and 25 $\frac{3}{8}$ % to decimals.

a) $\frac{1}{2}\ \% = \frac{\frac{1}{2}}{100} = \frac{1}{200} = 0.005$

b) $6.6\ \% = \frac{6.6}{100} = 0.066$

c) $0.003\ \% = \frac{0.003}{100} = 0.00003$

d) $25\ \frac{3}{8}\ \% = \frac{25.375}{100} = 0.25375$

In changing a mixed-number per cent to a decimal as in example d), the fractional part ($\frac{3}{8}$) is changed to equivalent decimal (0.375), added to the whole number (25) and divided by 100, dropping the sign %.

4.4 Changing Per Cents to Common Fractions

To change per cents to common fractions express the per cent as hundredths and reduce to lowest terms.

Examples: Change 6 %, 7 $\frac{1}{2}$ %, 5.5 % and 225 % to common fractions.

a) $6\ \% = \frac{6}{100} = \frac{3}{50}$

b) $7\ \frac{1}{2}\ \% = \frac{7.5}{100} = \frac{15}{200} = \frac{3}{40}$

c) $5.5\ \% = \frac{5.5}{100} = \frac{55}{1000} = \frac{11}{200}$

d) $225\ \% = \frac{225}{100} = 2\ \frac{25}{100} = 2\ \frac{1}{4}$

If a per cent is greater than 100, as in example d), it is changed to a whole number or a mixed number.

4.5 Rate, Base and Percentage

There are three elements present in problems concerning percentage. These elements are the BASE (b), the RATE (r) and the PERCENTAGE (p).

The base is the number to which the per cent is applied. The word which expresses this application is usually "of".

Thus:

$$\underbrace{25\ \%}_{\text{RATE (r)}} \text{ of } \underbrace{200}_{\text{BASE (b)}} = \underbrace{50}_{\text{PERCENTAGE (p)}}$$

200 is the base. The rate corresponds to the per cent. In the above statement it is 25 %. With the application of the rate to the base, the result is the third element, which is the percentage.

It can be observed, that the percentage is a product. Therefore "of" is used to express multiplication. It follows that: percentage equals rate times base.

4.6 Given the Base and the Rate to find the Percentage: p = br

To find the percentage, express the rate as a decimal fraction or a common fraction and multiply the base by this fraction.

Example: Find 6 % of 75.

Solution:

Rate (r) = 6 %	
$6\ \% = \frac{6}{100} = 0.06$	The rate, 6 % is expressed as decimal.
Base (b) = 75	The base is 75.
p = br	Substitute in the equation p = br to get the percentage 4.5
p = 75 x 0.06	
p = 4.5	

4.7 Given the Base and the Percentage to find the Rate

The ratio of the percentage to the base is the rate. We are required to express this ratio as a per cent.

In the equation $p = br$, b and r are the factors of the product (p), hence to obtain the rate (r) we divide the percentage (p) by the base (b) and express the quotient as a per cent.

Using symbols: $r = \frac{p}{b}$

Example a) What per cent of 20 is 5 ?

Solution:

$p = 5 \quad b = 20$	Percentage (p) is 5. Base (b) is 20.
$r = \frac{p}{b}$	Apply the equation $r = \frac{p}{b}$, to get the rate.
$r = \frac{5}{20} = 0.25$	
Express as per cent	Express quotient as a per cent.
$0.25 = \frac{25}{100} \times 100 = \underline{25\ \%}$	

Example b) What per cent of a foot is $\frac{1}{8}$ of an inch ?

Solution:

$p = \frac{1}{8}$ of an inch In decimal, $p = 0.125$ inch	The percentage (p) is $\frac{1}{8}$ of an inch.
$b = 1$ foot $= 12$ inches	There are 12 inches in a foot.
$r = \frac{p}{b}$ $r = \frac{0.125}{12}$	Apply the equation $r = \frac{p}{b}$.
$r = 0.0104 = \frac{104}{10000} \times 100 = \underline{1.04\ \%}$	Express quotient as a per cent.

4.8 Given the Percentage and the Rate to find the Base

It has been said, that the rate (r) and the base (b) are the two factors of the percentage (p). Hence either factor is equal to (p) divided by the other.

Therefore, $b = \frac{p}{r}$.

Example a) 75 is 50 % of what number ?

Solution:

$p = 75$	Percentage (p) is 75.
$r = 50\,\%$	
change to decimal	The rate (r), 50 % is expressed in decimal.
$50\,\% = \frac{50}{100} = 0.5$	
$b = \frac{p}{r} = \frac{75}{0.5}$	Apply the equation $b = \frac{p}{r}$,
$b = \underline{150}$	to get the base (b).

Example b) 150 is 50 % more than what number ?

The number (base) contains 100 % of itself. If 150 is 50 % more than the number, it is 150 % of the number.

Solution:

$p = 150$	Percentage is 150.
$r = 100\,\% + 50\,\% = 150\,\%$	The rate is 50 % more than the
change to decimal	number (base).
$150\,\% = \frac{150}{100} = 1.5$	Therefore it is 150 % of the
	number (base).
$b = \frac{p}{r} = \frac{150}{1.5}$	
$b = \underline{100}$	

Exercises:

1) Supply the missing items in Tables 1 and 2

TABLE 1				TABLE 2			
	Per Cent	Decimal	Common Fraction Whole or Mixed No.		Per Cent	Decimal	Common Fraction Whole or Mixed No.
a)	20%	____	____	a)	____	____	$9\frac{1}{2}$
b)	____	0.50	____	b)	____	____	$\frac{2}{3}$
c)	____	____	$\frac{7}{8}$	c)	____	1.75	____
d)	____	1.50	____	d)	____	5.05	____
e)	6%	____	____	e)	$4\frac{1}{4}$%	____	____
f)	$1\frac{3}{8}$%	____	____	f)	200%	____	____
g)	120%	____	____	g)	____	____	$\frac{1}{4}$
h)	____	0.05	____	h)	____	____	$\frac{15}{3}$
i)	____	____	$1\frac{1}{2}$	i)	____	0.02	____
j)	____	____	$\frac{1}{10}$	j)	____	0.33	____
k)	____	1.0	____	k)	____	____	$\frac{4}{9}$
l)	____	0.38	____	l)	____	____	$\frac{1}{10}$
m)	0.2%	____	____	m)	____	____	$\frac{1}{7}$
n)	$1\frac{2}{3}$%	____	____	n)	____	0.01	____
o)	0.6%	____	____	o)	$\frac{1}{6}$%	____	____
p)	____	0.0125	____	p)	30.2%	____	____
q)	____	0.52	____	q)	____	0.38	____

2) Find the percentage from these given rates and bases.

a) 12 % of 48

b) 55 % of 37

c) 21 % of 125

d) 28 % of 100

e) 95 % of 28

f) $12\frac{1}{2}$ % of 786

g) $83\frac{1}{3}$ % of 240

h) $\frac{3}{7}$ % of 420

i) $2\frac{1}{8}$ % of 96

j) 3.08 % of 832

3) Find the rates.

a) 20 is ______% of 100

b) 3 is ______% of 33

c) 56 is ______% of 70

d) 21 is ______% of 65

e) 100 is ______% of 200

f) What per cent of 9.8 is 4.6 ?

g) What per cent of $5\frac{1}{2}$ is $3\frac{1}{4}$?

h) What per cent of 7.2 is 18.5 ?

i) ______% of 3.6 is 2.8

j) ______% of 6 is 4.5

4) Find the base.

a) 8 = 33 % of ______

b) 150 % of ______ = 30

c) 35 = 25 % of ______

d) 10 = $2\frac{1}{2}$ % of ______

e) 25 is 25 % of ______

f) 48 is 20 % less than ______

g) 30 is $16\frac{2}{3}$ % more than ______

h) 22.4 is 65 % of ______

i) 25 is 20 % less than ______

j) 120 is 20 % more than ______

5) The per cent efficiency of a machine is found by dividing the output by the input and multiplying with hundred.

a) Input 5 KW Output 4.85 KW. Find efficiency in percentage.

b) Output $21\frac{1}{2}$ KW Input 22 KW. Find effeciency in percentage.

c) Output 25 KW Efficiency 75%. Find input in percentage.

d) Input $7\frac{1}{2}$ KW Output 7.35 KW. Find efficiency in percentage.

e) Input 50 KW Efficiency 90%. Find output in percentage.

Problems:

(6) The efficiency of a motor is 90 %; that is, the output is 90 % of the input. If the motor delivers 8 kW, what is the input ?

(7) In making up 95 kg of solder, 11 $\frac{1}{2}$ kg of lead and 83 $\frac{1}{2}$ kg of tin were used.
What per cent of each was used ?

(8) Aluminium weighs 70 % less than iron.
If an aluminium kettle weighs 0.33 kg, what would an iron kettle weigh, if it had the same volume of material ?

(9) A motor receiving 8 kW delivers 6.7 kW of work.
What per cent is the output of the input ?

(10) What per cent of a km are 120 m ?

(11) Out of a total production of 2715 bolts manufactured during a day, 107 were rejected by the quality control. What per cent of the total was rejected ?

(12) An alloy of common yellow brass is made of the following ingedients: copper, 170.5 kg; lead, 7.7 kg; tin, 0.55 kg; zinc, 96.25 kg. What per cent of the entire alloy does each of the metals represent ?

(13) In a gear train used to reduce speed, 4 % of the power supplied is lost in friction. If the power lost is found to be 2 $\frac{1}{2}$ kW, what is the power supplied ?

(14) A milling machine with 81 % efficiency has a loss of 1.43 kW. What is the power on the cutter ?

(15) The slip occurs in belt drives.
When there is no slip, the driving pulley rotates at 350 Rpm and the driven pulley at 250 Rpm.
If the slip is 1 %

A) what would be the speed of the driven pulley ?

B) what would be the amount of Rpm reduced ?

CHAPTER 5' ANGLE CALCULATION

5.1. Angle and its Units

When two lines meet at a point or cut each other at a point the angle is formed. (Fig. 5.01)

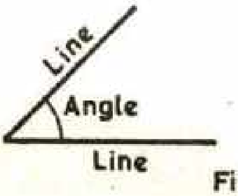


Fig. 5,01

The lines AB and CD -when produced- meet each other at a point "O", the angle AOC at point "O" is formed. (Fig. 5.02 and 5.03)

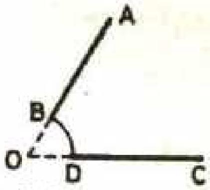


Fig. 5,02

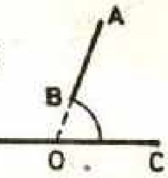


Fig. 5,03

When two lines, AB and CD cut each other at a point "O", four angles are formed, namely,

- α (alpha),
- β (beta),
- γ (gamma),
- θ (theta).

These are Greek names for the angles. (Fig. 5.04)

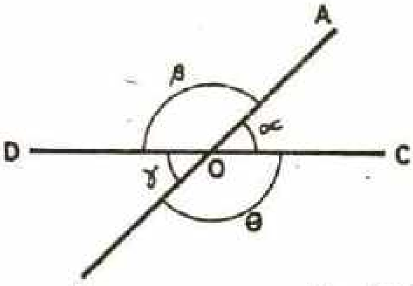


Fig. 5,04

When a circle is subscribed, the point which subscribes the circle is said to have travelled through a full angle. In other words, the circumference of a circle is called full angle. (Fig. 5.05)

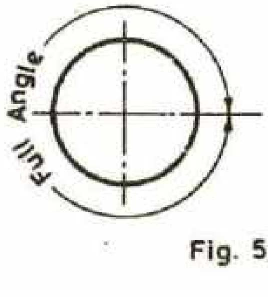


Fig. 5,05

The circumference is divided into 360 equal parts. Each part is called one degree (1°). When the circle is divided into four equal parts -quadrants-, each part consists of 90°. The first quadrant is from 0°-90°, the second from 90°- 180°, the third from 180°-270° and the fourth from 270°-360° (Fig. 5.06)

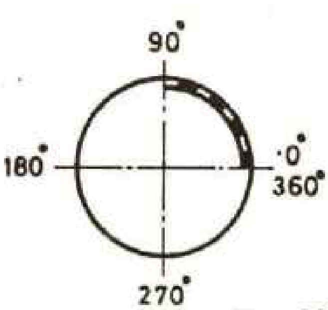


Fig. 5,06

The values of angles are always mentioned in degrees, minutes and seconds.

1 degree (1°) $= \frac{\text{circumference}}{360^\circ}$ (Fig. 5.07)

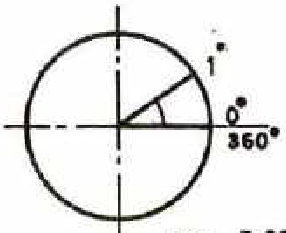


Fig 5,07

A degree is divided into minutes and seconds as follows:

1° = 60 minutes (60')

1' = 60 seconds (60")

5.2 Types of Angles

a) the angle which has more than 0° but less than 90° is called <u>acute angle</u>. (Fig. 5.08)

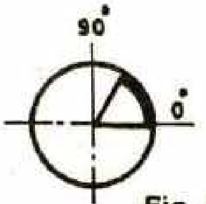


Fig. 5,08

b) the angle which has 90° is called <u>right angle</u>. (Fig. 5.09)

Fig. 5,09

c) the angle which has more than 90° but less than 180° is called <u>obtuse angle</u>. (Fig. 5.10)

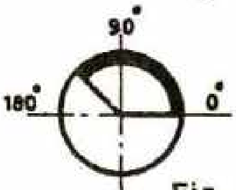


Fig. 5,10

d) the angle which has 180° is called <u>straight angle</u>. (Fig. 5.11)

Fig. 5,11

e) the angle which has more than 180° but less than 360° is called <u>reflex angle</u>. (Fig. 5.12)

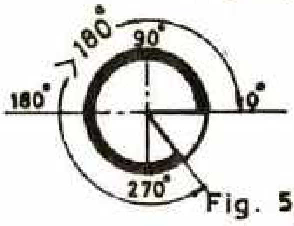


Fig. 5,12

f) the angle which has 360° is called <u>full angle</u>. (Fig. 5.13)

Fig. 5,13

5.3 Calculating Angle Values

Addition

First write the terms systematically, i.e. degrees under degrees, minutes under minutes, seconds under seconds.

Start adding from the lowest unit of angle towards the bigger in the same way as the addition of whole numbers.

Example:

Add $54^{\circ}55'$ and $24^{\circ}47'$

The result is $79^{\circ}42'$.

(Fig. 5.14)

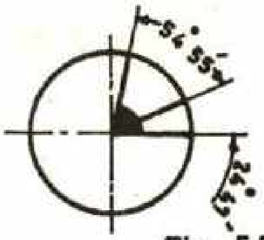


Fig. 5,14

When the two terms were added just like the addition of whole numbers, the result comes to $78^{\circ}102'$.

$$\begin{array}{r} 54^{\circ}\ 55' \\ \underline{24^{\circ}\ 47'} \\ 78^{\circ}102' \end{array}$$

Since 60' are equal to 1°, therefore the possible degrees should be made from 102' and be added to 78°. Thus 102' are equal to $1^{\circ}42'$.

Add $1^{\circ}42'$ to 78°.

$$102' = 1^{\circ}42'$$

$$78^{\circ} + 1^{\circ}42'$$

$$= \underline{79^{\circ}42'}$$

Subtraction

Write the terms systematically as in addition operation and proceed subtracting in the same way as in subtracting of whole numbers, i.e.:

$$\begin{array}{r} 156^{\circ}42'12" \\ -\ \underline{35^{\circ}32'10"} \\ 121^{\circ}10'02" \end{array}$$

Example:

Subtract $67^{\circ}35'$ from $137^{\circ}15'$

The result is $69^{\circ}40'$.

(Fig. 5.15)

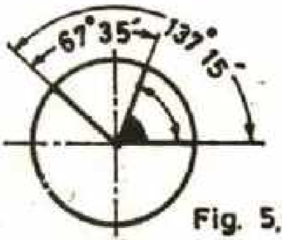


Fig. 5,15

$$\begin{array}{r} 137^{\circ}15' \\ -\ \underline{67^{\circ}35'} \end{array}$$

It is seen that in the minuend term the partial minuend 15' is smaller than the partial subtrahend 35'. Therefore 1° out of 137° should be converted in minutes and added to the partial minuend 15' thus bringing it to 75'.

Now we can subtract $67^{\circ}35'$ from $136^{\circ}75'$.

$$1^{\circ} = 60'$$

$$1^{\circ}+15' = 60'+15' = 75'$$

$$\begin{array}{r} 136^{\circ}75' \\ -\ \underline{67^{\circ}35'} \\ \underline{69^{\circ}40'} \end{array}$$

Multiplication

The multiplication starts from the lowest unit of the angle towards the higher one.

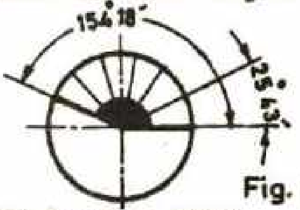


Fig. 5,16

Example:

Multiply 25°43' by 6

The result is 154°18'. (Fig.5.16)

Multiply 43' by 6.	43' x 6 = 258'
Convert 258' into degrees and minutes.	258' = 4°18'
Multiply 25° by 6.	25° x 6 = 150°
Add 4°18' and 150°.	4°18'+150° = 154°18'

Division

The division starts from the highest unit of the angle towards the lower one.

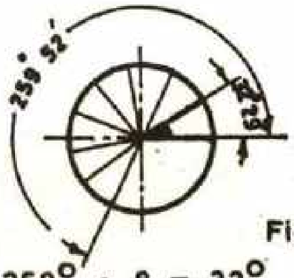


Fig. 5,17

Example:

Divide 259°52' by 8

The result is 32°29'. (Fig.5.17)

First divide 259° by 8. The quotient comes 32o and the remainder 3°.	259° : 8 = 32° Rem. 3°
Convert the remainder 3° into minutes.	3° = 180'
Add the partial dividend 52' and the remainder 180'.	52' + 180' = 232'
Divide 232' by 8. The quotient is 29'.	232' : 8 = 29'
Write the quotients together.	32° + 29' = <u>32°29'</u>

5.4 Expressing the Value of Angle in Decimal of Degree

The value of angles can be expressed in the decimal form of a degree as follows:

Examples:

a) Convert 48°31' into decimals of a degree

$48^\circ 31' = 48\frac{31}{60}$ $(1^\circ = 60')$

$\frac{31}{60} = 0.5166$ (converting $\frac{31}{60}$ to a decimal)

$48^\circ 31' = \underline{48.517^\circ}$ (correct to three places of decimals)

b) Convert 25°25'35" into decimals of a degree

$25'35" = 25 \times 60 + 35 = 1535"$ $(1^\circ = 60 \times 60 = 3600")$

$25^\circ 25'35" = 25\ \frac{1535}{3600}$ degrees

$= \underline{25.427^\circ}$ (converting $\frac{1535}{3600}$ to a decimal)

Exercises:

1) $29^{\circ}11' + 39^{\circ}51'$

2) [illegible]$5^{\circ}36' + 12^{\circ}35'$

3) $45^{\circ}0'13'' + 23^{\circ}47'55''$

4) $49^{\circ}51'39'' - 35^{\circ}45'35''$

5) $129^{\circ}33'55'' - 121^{\circ}39'57''$

6) $80^{\circ} - 71^{\circ}13'55''$

7) $145^{\circ}31'33'' - 139^{\circ}38'57''$

8) $12^{\circ}15' \times 5$

9) $45^{\circ}20'10'' \times 4$

10) $25^{\circ}30'25'' \times 7$

11) Divide the circle into 8 equal parts and give the value of each part.

12) Divide 3/4 of a circle into 5 equal parts and give the value of each part.

13) Divide $346^{\circ}2'$ into 7 equal parts and state the value of each part.

14) Convert the following values of angles into decimals of a degree:

a) $34^{\circ}45'$ b) $45^{\circ}36'56''$ c) $58^{\circ}57'19''$

CHAPTER 6 TOLERANCES

6.1 Tolerance in the dimensions is given to fix the limits in accuracy during manufacturing.

For example a shaft is required to be turned to a diameter of 10 mm within the accuracy limits of ± 0.05 mm. This shows that the nominal size of the shaft is 10 mm Ø and the limit of accuracy is fixed between the permitted maximum and minimum sizes. Fig. 6.01.

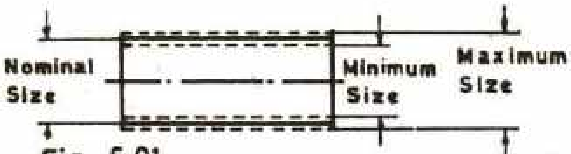


Fig. 6,01

Nominal size : Ø 10 mm
Maximum size : Ø 10.05 mm
Minimum size : Ø 9.95 mm

Tolerance is calculated by subtracting the minimum size from the maximum size.

tolerance = maximum size - minimum size

6.2 Bilateral and Unilateral Systems

When the limit of accuracy is mentioned towards both the sides of the nominal size, this system of fixing the limits of accuracy is called the Bilateral system as shown in Fig. 6.01 above.

When the limit of accuracy is mentioned towards one side of the nominal size, i.e. either towards upper or lower side depending upon the nature of fit, this system is called the Unilateral system as shown in Fig. 6.02 and 6.03.

Fig. 6,02

for
shaft shown in Fig. 6.02
nominal size = Ø 10 mm
limit of accuracy = + 0.05 mm
minimum size = Ø 10 mm
maximum size = Ø 10.05 mm
tolerance = 10.05 - 10 = 0.05 mm

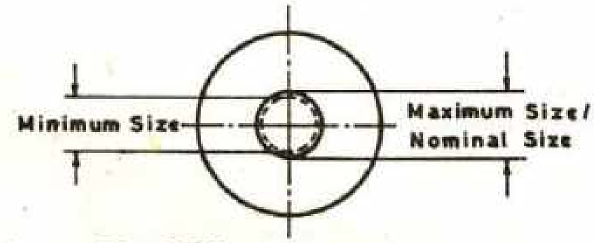


Fig 6,03

for
hole shown in Fig. 6.03
nominal size = Ø 10 mm
limit of accuracy = - 0.05 mm
minimum size = Ø 9.95 mm
maximum size = Ø 10 mm
tolerance = 10 - 9.95 = 0.05 mm

6.3 Actual Size

The dimension obtained during measuring is called the **actual size**.

For example the shaft in Fig. 6.01 was required to be turned within the accuracy limits of ± 0.05 mm. In this case the maximum size was 10.05 mm ø and the minimum size was 9.95 mm ø. But during the manufacturing the diameter of the shaft is measured to be 9.98 mm, which is between the permitted maximum and minimum sizes. Therefore the actual size of the shaft is 9.98 mm ø. Fig. 6.04.

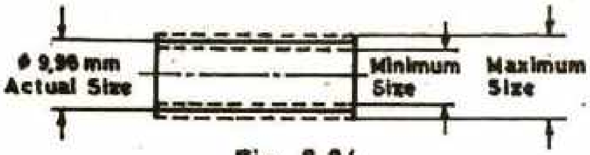


Fig. 6,04

Exercises:

1) Supply the missing items in the table.

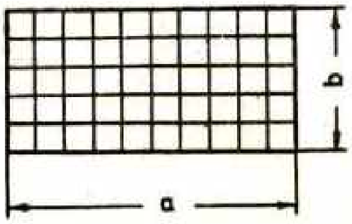
	Nominal size	Accuracy limits	Max.size	Min.size	System
a)	20	± 0.1			bilateral
b)	35			34.8	unilateral
c)		± 0.05	18.05		
d)		+ 0.2		16.00	
e)	40	± 0.1			
f)			10.1	9.95	
g)	18	+ 0.05			unilateral
h)		+ 0.1 - 0.05		39.95	
i)	55	± 0.1			

2) Which of the following actual sizes are within the accuracy limits?

	Actual size	Dimension in the Drawing	Within accuracy limits Yes/No
a)	19.98	20 ± 0.2	
b)	5.01	5 - 0.01	
c)	45.02	45 ± 0.3	
d)	29.98	30 + 0.2 - 0.1	
e)	64.00	64 + 0.2	

CHAPTER 7 SURFACE AREA (non-composed)

The formulae for calculating the surface areas of different plane engineering figures can be deduced both arithmetically and by means of drawings.



Area = 50 squares

If we draw a plane engineering figure, e.g. a rectangle on a squared paper, then the area of the rectangle is represented by the number of squares enclosed by it.

When counting the number of squares we realize that this can also be found by multiplying the sides of the rectangle. This gives us the formula for calculating a rectangle.

$$\boxed{\text{Area (rectangle)} = \text{side a} \times \text{side b}}$$

7.1 Units of area

The metric units of area such as square millimetre, square centimetre and square metre are derived from the international units of linear measurement, i.e. millimetre, centimetre and metre.

The following are the metric units of measuring area:

square metre = m^2 $1\ m^2 = 10000\ cm^2$

square centimetre = cm^2 $1\ cm^2 = 100\ mm^2$

square millimetre = mm^2 $1\ mm^2 = 0.01\ cm^2$

Remark: The previously used British units, e.g. square inch, square foot, have been replaced by metric units.
1 square inch = $6.452\ cm^2$

7.2 Powers

When we say that a certain number is raised to some power, it means that the number is multiplied by itself as many times as the power to which it is raised.

4×4 .... is called four to the power of two or four squared. It is written as .... 4^2.
$= 4^2$

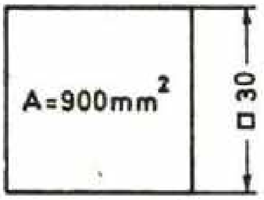
$4 \times 4 \times 4$ .... is called four to the power of three or four cubed. It is written as .... 4^3.
$= 4^3$

$4 \times 4 \times 4 \times 4$ ...... is called four to the power of four or four to the fourth power, written .... 4^4.
$= 4^4$

Note: The figure that indicates the power to which a number is to be raised is called the index, e.g. the index of 4^3 is 3.

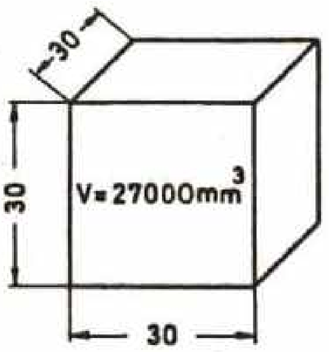
The problem can also be demonstrated by means of a drawing.

When drawing a number to the power of two we are actually drawing an area, thus squaring the length of one side.



$$\text{Area (A)} = \text{one side to the power of two}$$
$$= \text{one side squared} = S^2$$
$$= (30 \text{ mm})^2 = 30 \text{ mm} \times 30 \text{ mm} = \underline{900 \text{ mm}^2}$$

If we raise one side to the power of three, we are actually calculating the volume (V) of a cube.



$$\text{Volume (V)} = \text{one side to the power of 3}$$
$$= \text{one side cubed} = S^3$$
$$= (30 \text{ mm})^3 = 30\text{mm} \times 30\text{mm} \times 30\text{mm}$$
$$= \underline{27000 \text{ mm}^3}$$

Note: When raising a length to some power (e.g. the side of a square) the number and its units have to be raised. Whereas the number may be calculated now, its unit remains with the index.

7.3 Formulae for calculating surface area

Symbols used in calculation:

a, b, c = length of sides

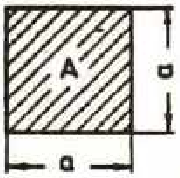
h = perpendicular height

d, D = diameter

A = surface area

Square

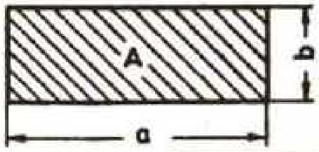
A square has four equal sides at right angles.



Formula: $\boxed{A = a \times a}$

Rectangle

A rectangle has four sides at right angles. The opposite ones are always equal.

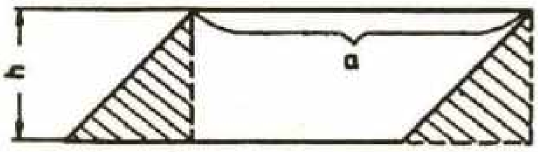


b = h

Formula: $\boxed{A = a \times b = a \times h}$

Parallelogram

A parallelogram has four sides. The opposite sides are equal and run parallel. In case of a rhombus all sides are equal.

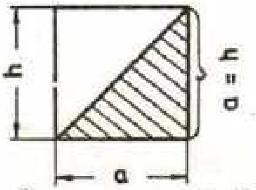


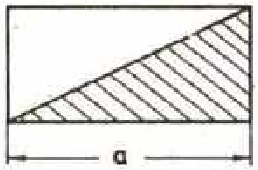
Formula: $\boxed{A = a \times h}$

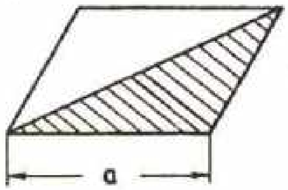
Note: As the surface of the parallelogram is equal to that of a rectangle, it is calculated in the same way.

Triangle

A triangle consists of three sides. It can be formed out of squares, rectangles and parallelograms.







$A_{square} = a \times h$

$A_{triangle} = \frac{a \times h}{2}$

$A_{rect.} = a \times h$

$A_{triangle} = \frac{a \times h}{2}$

$A_{parall.} = a \times h$

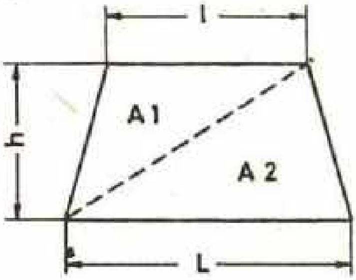
$A_{triangle} = \frac{a \times h}{2}$

Formula: $\boxed{A_{triangle} = \frac{a \times h}{2}}$

Trapezium

A trapezium has four sides out of which two run parallel.

To find the surface area, we divide the trapezium into two triangles, calculate their surface areas and add them together.



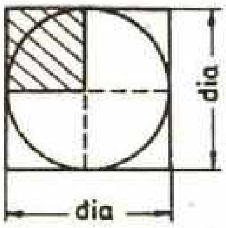
$A_1 = \frac{l \times h}{2}$ $A_2 = \frac{L \times h}{2}$

$A_{trapezium} = \frac{L \times h}{2} + \frac{l \times h}{2}$

Formula: $\boxed{A_{trapezium} = \frac{L + l}{2} \times h}$

Circle

A circle can be defined as a regularly bent line all of whose points are at an equal distance (radius) fr the centre point.



Deducing the formula by means of a drawing can only be done with a limited accuracy.

It has, however, been found out that the area of a circle can be calculated by multiplying the small square by the number 3.14.
As this is to be done for the calculation of all circles, this number has been given a special symbol: $3.14 = \pi$ (pronounced: pee)

$d \times d = A_{big\ sq.}$

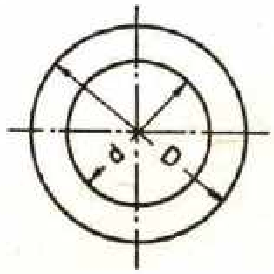
$\frac{d \times d}{4} = A_{small\ sq.}$

Formula: $\boxed{A_{circle} = \frac{d \times d}{4} \times 3{,}14 \quad \underline{or} = \frac{d^2 \times \pi}{4}}$

$A_{circle} = 3{,}14 \times A_{small\ sq.}$

Annulus

An annulus consists of two circles with different diameters.



For calculating the surface area we subtract the areas of the circles from each other:

$$A_{annulus} = A_{big\ dia} - A_{small\ dia}$$

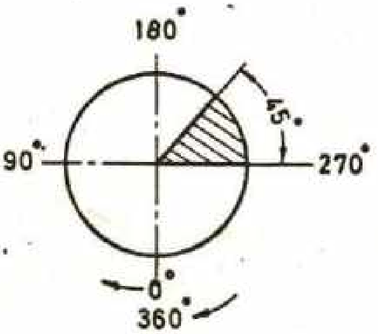
Formula: $A_{annulus} = \frac{D^2 \times \pi}{4} - \frac{d^2 \times \pi}{4}$

$$\boxed{A = (D^2 - d^2) \times \frac{\pi}{4}}$$

Sector of a circle

A sector of a circle is a fraction of a whole circle.

To find the surface area we multiply the circle-area by the fraction of the circle sector.



The regularly bent line of a circle can be parted in units, called degrees.

A whole circle has 360 degrees = 360°.

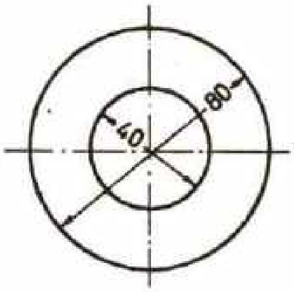
$$A_{sector} = A_{circle} \times \text{fraction}$$

Formula: $\boxed{A_{sector} = \frac{d^2 \times \pi}{4} \times \frac{\alpha}{360^\circ}}$

Examples

a) Calculate the amount of sheet metal required for producing the bearing cover? (in cm^2)

Given: $D = 80\ mm = 8\ cm$
$d = 40\ mm = 4\ cm$



Solution: $A = \frac{D^2 \times \pi}{4} - \frac{d^2 \cdot \times \pi}{4}$

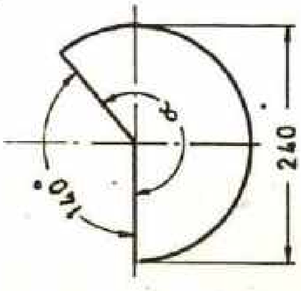
$= \frac{8^2 \times \pi}{4} - \frac{4^2 \times \pi}{4}$

$= 50.26 - 12.56$

$= \underline{37.70\ cm^2}$

b) How much sheet metal will be needed for producing the protection cover of a pedestal grinder? (in cm^2)

Given: $d = 240\ mm$
$= 220^\circ$

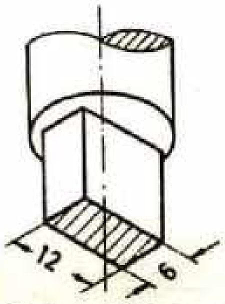


Solution: $A = \frac{d^2 \times \pi}{4} \times \frac{\alpha}{360^\circ} = \frac{24^2 \times \pi}{4} \times \frac{220^\circ}{360^\circ}$

$= \underline{276{,}1\ cm^2}$

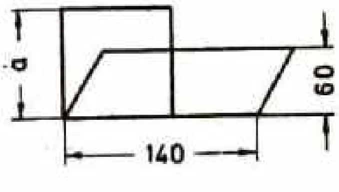
Exercises

1)



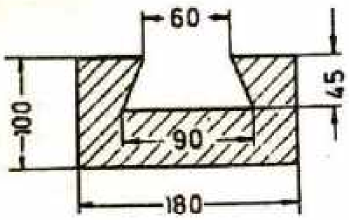
Calculate the surface area A of the punch-face.

2)



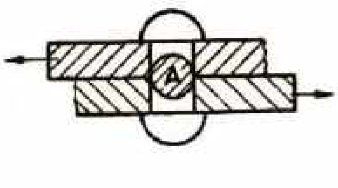
Find the dimensions of the square (with sides 'a') whose surface area is equal to the surface area of the parallelogram.

3)



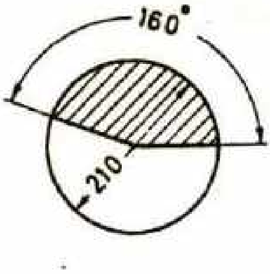
Calculate the cross-section of the dovetail-guide.

4)



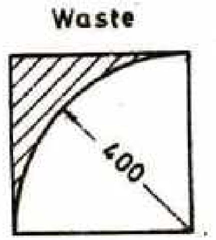
The dia of the closed rivet is 21 mm. Calculate its cross-section.

5)



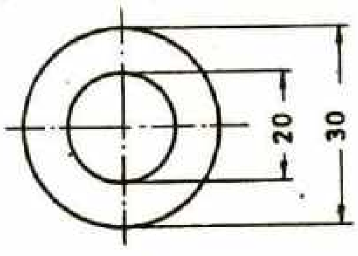
A funnel will be produced out of sheet-metal. Calculate the amount of metal required (in cm^2).

6)



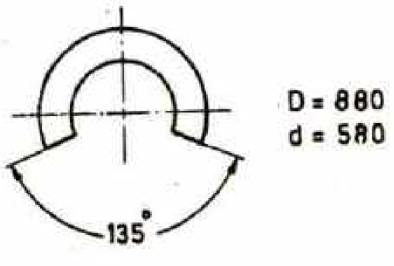
Calculate the wastage in % when cutting the cover sheet.

7)



The surface area for the washer is required.

8)



Calculate the area of the sector.

CHAPTER 8 USE OF TABLES

In order to simplify the work for the most common types of calculations, tables have been developed.
With the table given below we can find:

a) with a given base number (n) or dia (d) the square, square root, circumference and circle area.
b) with a given square, square root, circumference or circle area the base number or diameter.

d or n	n^2	$\sqrt{n}$	$d \cdot \pi$	$\frac{d^2\pi}{4}$
0,1	0,01	0,3162	0,314	0,0079
0,2	0,04	0,4472	0,628	0,0314
0,3	0,09	0,5477	0,942	0,0707
0.4	0,16	0,6325	1,257	0,1257
0,5	0,25	0,7071	1,571	0,1964
0,6	0,36	0,7746	1,885	0,2827
0,7	0,49	0,8367	2,199	0,2848
0.8	0,64	0,8942	2,513	0,5025
0,9	0,81	0,9487	2,827	0,6362
1,0	1,00	1,0000	3,142	0,7854
2	4	1.4142	6,283	3,1416
3	9	1,7321	9,425	7,0686
4	16	2,0000	12,566	12,57
5	25	2,2361	15,708	19,63
6	36	2,4495	18,850	28,27
7	49	2,6458	21,991	38,48
8	64	2.8284	25,133	50,27
9	81	3,0000	28,274	63,62
10	100	3,1623	31,416	78,54
11	121	3,3166	34,558	95,03
12	144	3,4641	37,699	113,10
13	169	3,6056	40,841	132,73
14	196	3.7417	43,982	153,94
15	225	3,8730	47,124	176,71
16	256	4,0000	50,265	201,06
17	289	4,1231	53,407	226,98
18	324	4,2426	56,549	254,47
19	361	4,3589	56,690	283,53
20	400	4.4721	62,832	314,16
21	441	4,5826	65,973	346,36
22	484	4,6904	69,115	380,13
23	529	4,7958	72,257	415,48
24	576	4,8990	75,398	452,39
25	625	5,0000	78,540	490,87
26	676	5.0990	81,681	530,93
27	729	5,1962	84,823	572,56
28	784	5,2915	87,965	615,75
29	841	5,3852	91,106	660.52
30	900	5,4772	94,248	706,86
31	961	5,5678	97,39	754,77
32	1024	5,6569	100,53	804,25
33	1089	5,7446	103,67	855,30
34	1156	5.8310	106,81	907,92
35	1225	5,9161	109.96	962,11
36	1296	6,0000	113,10	1017,88
37	1369	6,0828	116,24	1075,21
38	1444	6,1644	119,38	1134,11
39	1521	6,2450	122,52	1194,59
40	1600	6,3246	125,66	1256.64
41	1681	6,4031	128,81	1320,25
42	1764	6,4807	131,95	1385,44
43	1849	6,5574	135,09	1452,20
44	1956	6,6332	138,23	1520,53
45	2025	6,7082	141.37	1590,43
46	2116	6,7823	144,51	1661,90
47	2209	6,8557	147,65	1734,94
48	2304	6,9282	150,80	1809,56
49	2401	7,0000	153,94	1885,74
50	2500	7,0711	157,08	1963.50
51	2 601	7,1414	160,22	2 042,82
52	2 704	7,2111	163,36	2 123,72
53	2 809	7,2801	166,50	2 206,18
54	2 916	7,3485	169,65	2 290,22
55	3 025	7,4162	172.79	2 375,83
56	3 136	7,4833	175,93	2 463,01
57	3 249	7,5498	179,07	2 551,76
58	3 364	7,6158	182,21	2 642,08
59	3 481	7,6811	185,35	2 733,97
60	3 600	7,7460	188,50	2 827.43
61	3 721	7,8102	191,64	2 922,47
62	3 844	7,8740	194,78	3 019,07
63	3 969	7,9373	197,92	3 117,25
64	4 096	8,0000	201,06	3 216,99
65	4 225	8,0623	204,20	3 318,31
66	4 356	8,1240	207,35	3 421,19
67	4 489	8,1854	210,49	3 525,65
68	4 624	8,2462	213,63	3 631,68
69	4 761	8,3066	216,77	3 739,28
70	4 900	8,3666	219,91	3 848.45
71	5 041	8,4261	223,05	3 959,19
72	5 184	8,4853	226,19	4 071,50
73	5 329	8,5440	229,34	4 185,39
74	5 476	8,6023	232,48	4 300,84
75	5 625	8,6603	235.62	4 417,86
76	5 776	8,7178	238,76	4 536,46
77	5 929	8,7750	241,90	4 656,63
78	6 084	8,8318	245,04	4 778,36
79	6 241	8,8882	248,19	4 901,67
80	6 400	8,9443	251,33	5 026.55
81	6 561	9,0000	254,47	5 153,00
82	6 724	9,0554	257,61	5 281,02
83	6 889	9,1104	260,75	5 410,61
84	7 056	9,1652	263,89	5 541,77
85	7 225	9,2195	267.04	5 674,50
86	7 396	9,2736	270,18	5 808,80
87	7 569	9,3274	273,32	5 944,68
88	7 744	9,3808	276,46	6 082,12
89	7 921	9,4340	279,60	6 221,14
90	8 100	9,4868	282,74	6 361.73
91	8 281	9,5394	285,88	6 504
92	8 464	9,5917	289,03	6 648
93	8 649	9,6437	292,17	6 793
94	8 836	9,6954	295,31	6 940
95	9 025	9,7468	298,45	7 088
96	9 216	9,7980	301,59	7 238
97	9 409	9,8489	304.73	7 390
98	9 604	9,8995	307,88	7 543
99	9 801	9,9499	311,02	7 698
100	10 000	10,0000	314,16	7 854
101	10 201	10,0495	317,30	8 012
102	10 404	10,0995	320,44	8 171
103	10 609	10,1489	323,58	8 332
104	10 816	10,1980	326,73	8 495
105	11 025	10.2470	329,87	8 659
106	11 236	10,2956	333,01	8 825
107	11 449	10,3441	336,15	8 992
108	11 664	10,3923	339,29	9 161
109	11 881	10,4403	342,43	9 331
110	12 100	10,4881	345,58	9 503

1. **Square**

n	n^2
20	400

2. **Root**

n	n^2	$\sqrt{n}$
20	→	4,4721

3. **Circumference**

d	n^2	$\sqrt{n}$	$d \times \pi$
20		→	62,832

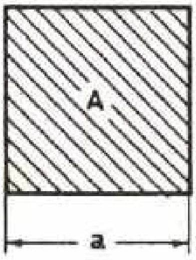
4. **Circle Area**

d	n^2	$\sqrt{n}$	$d\pi$	$\frac{d^2\pi}{4}$
20			→	314,16

Note:

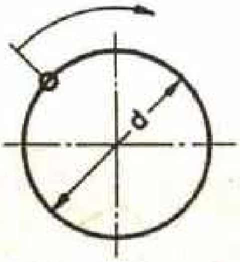
Further specifications may be looked up in a table book.

Exercises



1) Find from the table the area of the cross section of the following square steel bars:
□ 12 mm; □ 1,5 cm; □ 95 mm.

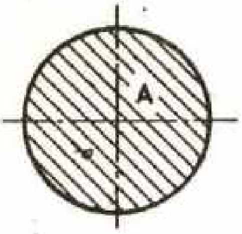
2) Find the length of side "a" when the square cross section is known.
A = 7225 mm^2; A = 529 cm^2; A = 4225 mm^2.



3) Calculate the circumference of round steel bars with the following diameters:
d = 15 cm; d = 82 mm; d = 105 mm.

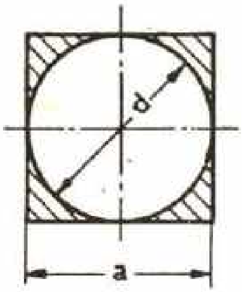
4) Find the diameter of a wheel when its circumference is 213,63 cm.

5) After rotating three times, a wheel has moved a distance of 339,3 cm.
Find the dia of the wheel.



6) Find from the table the surface area of the sheet disk
d = 42 mm; d = 5,6 cm.

7) Find the dia of the screws when the area of their cross section is given as
78,54 mm^2; 452,39 mm^2.



8) Calculate the wastage by subtracting the surface area of the steel disk from that of the square.
d = a = 84 mm; d = a = 108 cm.

9) Suppose the disk is welded on the steel sheet. Calculate the length of the butt weld.
(d as no. 8)

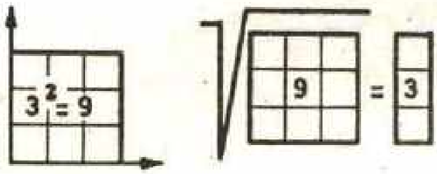
10) Find from the table a dia for a circle whose surface area comes closest to the surface area of the squares of no. 8.

CHAPTER 9 PYTHAGOREAN PROPOSITION

Remark: For dealing with the Pythagorean proposition the following mathematical problems have to be discussed beforehand:

/ Evolving of roots / Transposition of formulae

9.1 Evolving of Roots



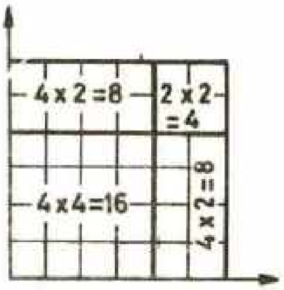
A root is converse to the power, i.e. we have to find out what number has been raised to a certain power.

The craftsman, generally, is only dealing with square roots of whole or decimal numbers.

The sign of the roots is $\sqrt[2]{}$. A small index figure is added to tell what power the root is. In the case of square roots the index may be left out: $\sqrt{}$

Method of evolving the square root

Background: In order to evolve a square root, the surface of a square has actually been parted as shown here:



Square I $4 \times 4 = 16 \rightarrow a^2$

Rectangle I and II $2 \times 4 \times 2 = 16 \rightarrow 2ab$

Square II $2 \times 2 = 4 \rightarrow b^2$

Sum of the part-surfaces: $36 = a^2 + 2ab + b^2$

$$\sqrt{36} = \sqrt{a^2 + 2ab + b^2}$$

Application:

The practical way of evolving of square roots is as follows:-

Example: Find the square root of 625 !

$\sqrt{625} = \sqrt{625.00}$

step 1: Set a decimal point and add two zeros thus making the number a decimal (only when it is not already a decimal number).

$\sqrt{6'25!00'}$

step 2: Divide the number into periods of two, beginning with the decimal point.

$\sqrt{6'25!00'} = 2$
$\underline{4}$
2

step 3: Find the largest square fitting into the first period. The root of this square forms the first part of the answer. Subtract the square from the first period.

$\sqrt{6'25!00'} = 2$
$\underline{4}$
$2\ 2(5) : 4 = 5$

step 4: Bring down the next period. Cover the last figure of the newly formed number. Divide by the doubled part-result. Enter this part result in the divisor.

$\sqrt{6'25!00'} = 25$
$\underline{4}$
$22(5) : 45$
$\underline{22\ 5}$

step 5: Multiply the newly formed divisor by part result of step 4 and subtract.
Mind: When the subtraction is not possible your last part-result was wrong. Do step 4 again.
Enter the new part-result in the final result.

Note: In case of larger numbers or remainders start from step no.4 anew.

Result:

$\sqrt{625} = 25$

Proof:

$25 \times 25 = 625$

Remark: Train also evolving of roots by using a table as given in part no. 1 of this textbook.

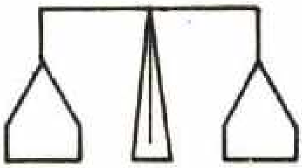
9.2 Transposition of Formulae

Formulae (repetition):

A formula is an equation. It can be compared with a scale. That means:

/To maintain the balance, both the sides of the formula (or equation, or scale) must have the same value.

/Having the same value means that the total of one side is equal to the total of the other side, whether or not the total value is expressed as a 'single' value or as a 'composed' value.



$24 = 24$

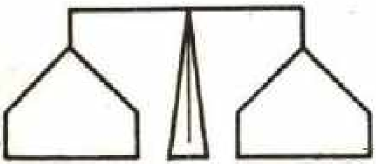
$24 = 2 \times 12$

$24 = 48 \div 2$

$24 = 19 + 5$

$24 = 27 - 3$

Having the same value also means that one side is equal to the other side, whether or not the value is expressed by a symbol, because symbols represent values.
Example: Suppose the length (L) of a bar is six times the length (a) of a shorter bar. Then we can form the following equations:



$L = 2 \times 3a$

$L = 12a \div 2$

$L = 4a + 2a$

$L = 9a - 3a$

Mind: $6a = 6 \times a = a \times 6$

Note: We can change the sides of a scale and still maintain the balance. We can also change the sides of the formula.

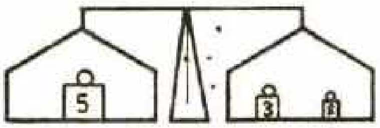
$5 \text{ kg} = 3 \text{ kg} + 2 \text{ kg}$

$3 \text{ kg} + 2 \text{ kg} = 5 \text{ kg}$

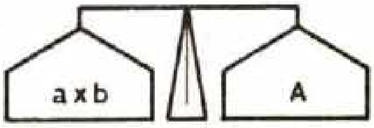
Transposition:

To transpose a formula means: To shift the quantities or symbols from one side to the other so that the required one is placed alone on one side. This transposition is to be done in a way so that the scale (or formula, or equation) still remains in balance.

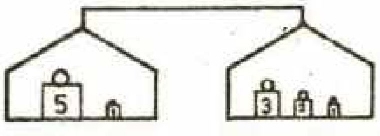
The scale will stay in balance as long as the same operations are done on both of its sides:



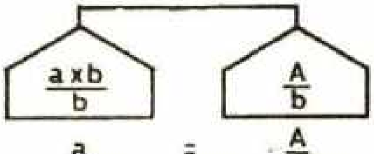
$5 \text{ kg} = 3 \text{ kg} + 2 \text{ kg}$



$a \times b = A$



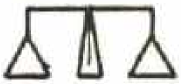
$5 \text{ kg} + 1 \text{ kg} = 3 \text{ kg} + 2 \text{ kg} + 1 \text{ kg}$



$a = \frac{A}{b}$

Transposition of Formulae:

Using multiplication and/or division



General Steps

$A = a \times b$

Step 1. Write down the formula to be used.

$A = a \times \underline{b}$

Step 2. Mark the quantity or symbol, the value of which has to be found.

Step 3. Remove all other quantities or symbols on the side of the symbol marked in step 2.
First, find the operation which is the opposite of the operation of the quantity to be removed.
Second, perform this operation on both sides.

$\frac{A}{a} = \frac{a \times \underline{b}}{a}$

Note: Multiplication is the opposite of division.
Division is the opposite of multiplication.

Mind: Both sides of the formula (like a scale) must remain equivalent (in balance).

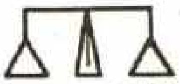
$\frac{A}{a} = \underline{b}$

Step 4. Simplify by cancellation (by performing the operations).
Re-write the formula.

$\underline{b} = \frac{A}{a}$

Example a)

Follow steps 1 to 4 verbally.



$A = \frac{a \times h}{2}$

Step 1. $\frac{a \times h}{2} = A$

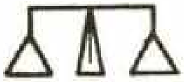
Step 2. $\frac{\underline{a} \times h}{2} = A$

Step 3. $\frac{\underline{a} \times h \times 2}{2 \times h} = \frac{A \times 2}{h}$

Step 4. $\underline{a} = \frac{A \times 2}{h}$

Example b)

Find the missing length "a"



$F \times a = b \times Q$

Step 1. $F \times a = b \times Q$

Step 2. $F \times \underline{a} = b \times Q$

Step 3. $\frac{F \times \underline{a}}{F} = \frac{b \times Q}{F}$

Step 4. $\underline{a} = \frac{b \times Q}{F}$

Transposition of Formulae:

Using addition and/or subtraction

General Steps

$a^2+b^2 = c^2$	Step 1. Write down the formula to be used.
$\underline{a}^2+b^2 = c^2$	Step 2. Mark the quantity or symbol, the value of which has to be found.
$\underline{a}^2+b^2-b^2 = c^2-b^2$	Step 3. Remove all other quantities or symbols on the side of the symbol marked in step 2. First, find the operation which is the opposite of the operation of the quantity to be removed. Second, perform this operation on both sides. Note: Addition is the opposite of subtraction. Subtraction is the opposite of addition. Mind: Both sides of the formula (like a scale) must remain equivalent (in balance).
$\underline{a}^2 = c^2-b^2$	Step 4. Simplify by cancellation (by performing the operations). Re-write the formula.

Example a)

Follow steps 1 to 4 verbally.

Step 1. $a+5 = b-10$

Step 2. $a+5 = \underline{b}-10$

Step 3. $a+5+10 = \underline{b}-10+10$

Step 4. $a+15 = \underline{b}$

<u>or:</u> $\underline{b} = a+15$

Example b)

Find the missing length "l_2"

Step 1. $L = l_1+l_2+l_3+l_4$

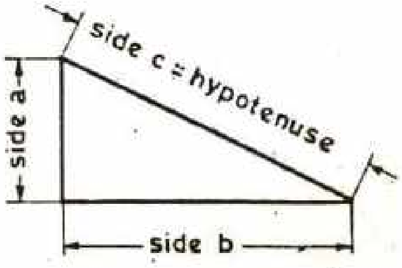
Step 2. $L = l_1+\underline{l_2}+l_3+l_4$

Step 3. $L -l_1-l_3-l_4 = l_1+\underline{l_2}+l_3+l_4 -l_1-l_3-l_4$

Step 4. $L -l_1-l_3-l_4 = \underline{l_2}$

or: $\underline{l_2} = L -l_1-l_3-l_4$

9.3 Pythagorean Proposition



Statement:

In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the sides.

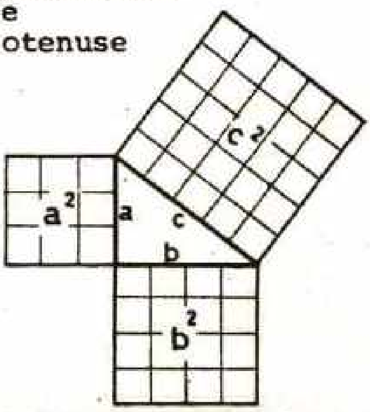
Side a = perpendicular
Side b = base
Side c = hypotenuse

Derivation of the Formulae:

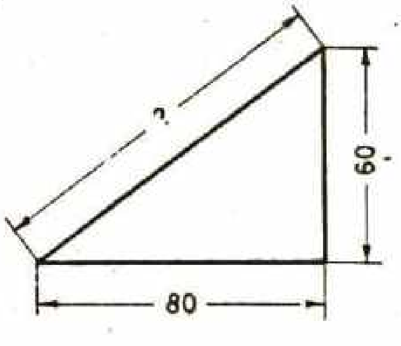
$c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$

$a^2 = c^2 - b^2$ or $a = \sqrt{c^2 - b^2}$

$b^2 = c^2 - a^2$ or $b = \sqrt{c^2 - a^2}$



Note: When the lengths of two sides of a right angled triangle are known, the length of the third one can be calculated.



Example a)

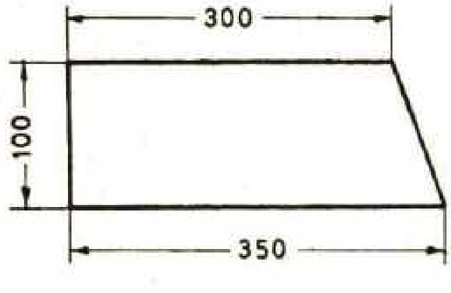
Calculate the length of the hypotenuse (side c) of the triangle shown on the left.

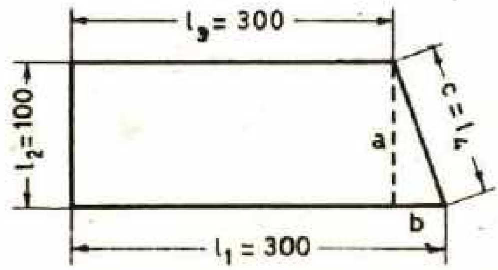
Given: $a = 60$ mm, $b = 80$ mm

Solution: $c^2 = a^2 + b^2 = 60^2 + 80^2$

$$c = \sqrt{60^2+80^2} = \sqrt{10{,}000}$$

$$c = \underline{100 \text{ mm}}$$





Example b)

Calculate the total length of the cut of the metal sheet.

Given: $l_1=350$, $l_2=100$, $l_3=300$

$a=l_2=100$, $b=l_1-l_3=350 - 300$

Mind: As shown in the second drawing, side "c" of the right angled triangle must be found before.

Solution: $L = l_1 + l_2 + l_3 + l_4$

$$l_4 = c = = \sqrt{100^2+50^2} = \sqrt{12{,}500}$$

$$L = 350+100+300+112 = \underline{862}$$

Exercises:

a) Evolve the roots

1) $\sqrt{324} =$

2) $\sqrt{6561} =$

3) $\sqrt{14.6} =$

4) $\sqrt{5^2} =$

5) $\sqrt{142^2} =$

6) $\sqrt{a^2} =$

7) $\sqrt{cm^2} =$

8) $\sqrt{16\ cm^2} =$

9) $\sqrt{987.5\ m^2} =$

---

b) Transpose the formulae

1) $3 \times a = 15$

$a = ?$

2) $\frac{a \times b}{2} = A$

$a = ?$

$b = ?$

3) $a + b = d$

$a = ?$

$b = ?$

4) $F \times a = Q \times b$

$F = ?$

$Q = ?$

5) $\frac{a \times h}{2} = A$

$a = ?$

$h = ?$

6) $A = \frac{d^2 \pi}{4}$

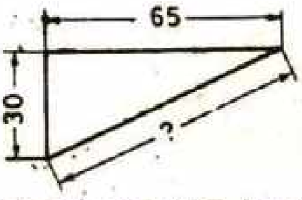
$d^2 = ?$

$d = ?$

---

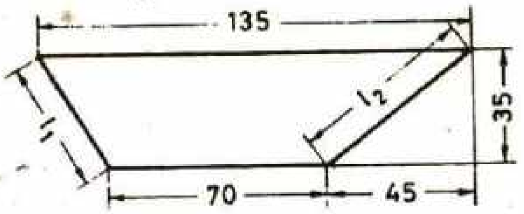
c) Use of Pythagorean Proposition

1)

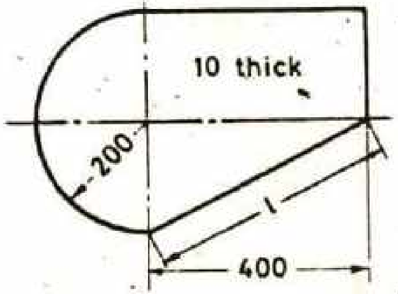


Calculate the length of the hypotenuse

2) Calculate the lengths "l_1" and "l_2"

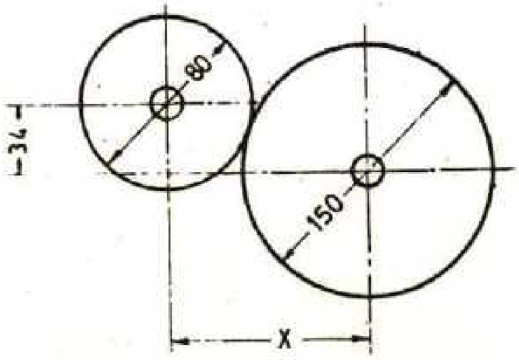


3)



Calculate the length "l" of the steel sheet

4) Calculate dimension "x".



CHAPTER 10

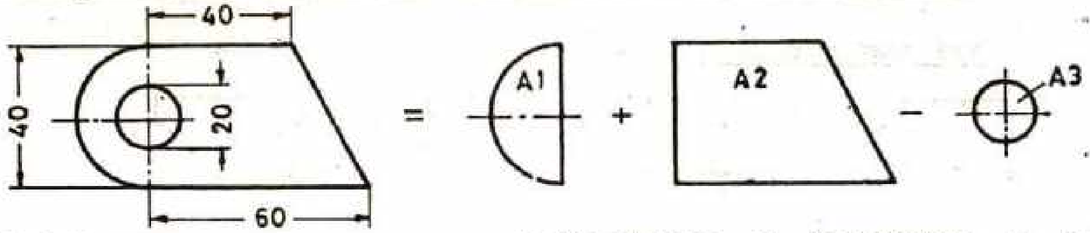
SURFACE AREA (composed figures)
PERIMETER - CIRCUMFERENCE

10.1 Composed Surfaces:

Every composed surface can be divided into simple or non-composed areas, i.e. square, rectangle, triangle, trapezium, circle, sector of a circle.

To calculate the surface area of composed figures, therefore, the first step will be the determination of the simple figures involved.

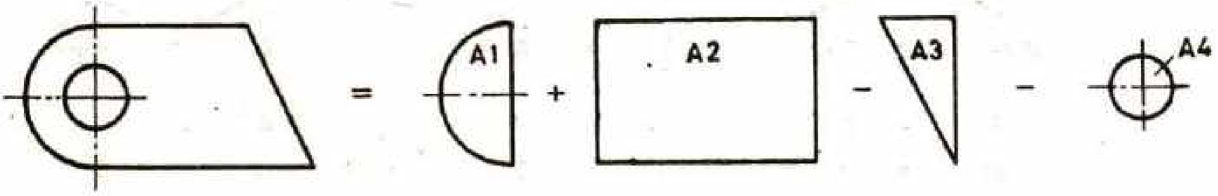
Example a)



Total area = half circle + trapezium - circle

In most cases the partition can be done in more than one way, e.g.

Example b)



The next step will be to set up the formula and calculate accordingly:

Example a)

$A = A_1 + A_2 - A_3$

$A_1 = \frac{d^2 x \pi}{4} : 2 = \frac{40^2 x 3.14}{4} : 2$

$A_1 = \underline{628 \text{ mm}^2}$

$A_2 = \frac{L + 1}{2} x \text{ h} = \frac{60 + 40}{2} x40$

$A_2 = \underline{2000 \text{ mm}^2}$

$A_3 = \frac{d^2 x \pi}{4} = \frac{20^2 x 3.14}{4} = \underline{314 \text{ mm}^2}$

$A = 628 + 2000 - 314$

$A = 2314 \text{ mm}^2$

========

Example b)

$A = A_1 + A_2 - A_3 - A_4$

$A_1 = \frac{d^2 x \pi}{4} : 2 = \frac{40^2 x 3.14}{4} : 2$

$A_1 = \underline{628 \text{ mm}^2}$

$A_2 = a \text{ x } b = 60 \text{ x } 40$

$A_2 = \underline{2400 \text{ mm}^2}$

$A_3 = \frac{a \text{ x } b}{2} = \frac{20 \text{ x } 40}{2} = \underline{400 \text{ mm}^2}$

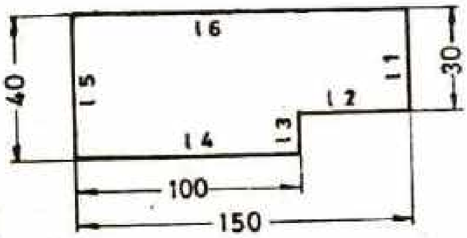
$A_4 = \frac{d^2 x \pi}{4} = \frac{20^2 x 3.14}{4} = \underline{314 \text{ mm}^2}$

$A = 628 + 2400 - 400 - 314$

$A = 2314 \text{ mm}^2$

========

10.2 Perimeter :

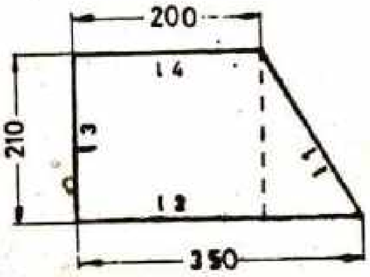


For calculating the perimeter of rectangular workpieces we simply add the lengths of all the sides:

$$L = l_1 + l_2 + l_3 + l_4 + l_5 + l_6$$

Note: $l_2 = l_6 - l_4 = 150 - 100 = 50$

$l_3 = l_5 - l_1 = 40 - 30 = 10$



Pythagoras:

$c = l_1;\ a = l_3$

$b = l_2 - l_4 = 150$

$l_1^2 = 210^2 + 150^2$

$l_1 = \sqrt{210^2 + 150^2}$

The perimeter of workpieces with inclined sides will be calculated similarily by adding the sides. In most cases the length of the inclined line can be calculated by utilizing the pythagorean proposition:

$$L = l_1 + l_2 + l_3 + l_4$$

$l_1 = \sqrt{210^2+150^2} = \sqrt{66600}$

$= \underline{291\ mm}$

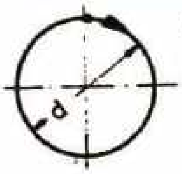
$l_2 = \underline{350\ mm}$

$l_3 = \underline{210\ mm}$

$l_4 = \underline{200\ mm}$

$L = 291 + 350 + 210 + 200 = \underline{\underline{1051\ mm}}$

10.3 Circumference:

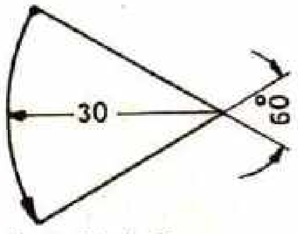


$L = d \times \pi$

The circumference of whole circular workpieces is calculated by multiplying the diameter by the number 3.14 (pee = π)

Formula:

$L = dia \times 3.14$ or: $L = d \times \pi$



$d = r + r$

$L = d \times \pi \times \frac{\alpha°}{360°}$

$= 60 \times 3,14 \times \frac{60°}{360°}$

$= 188.5\ mm \times \frac{1}{6} = 31.41\ mm$

========

To find the circumference of a sector of a circle we multiply the whole circumference by the fraction of the circle sector.

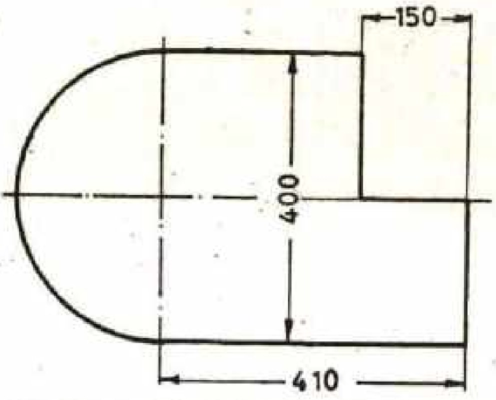
Formula:

$L = d \times \pi \times \frac{\alpha°}{360°}$

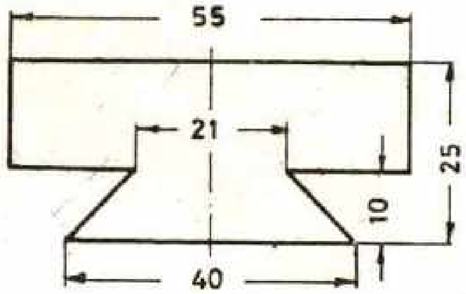
Exercises:

1) Calculate surface area (in cm^2) and perimeter / circumference (in cm).

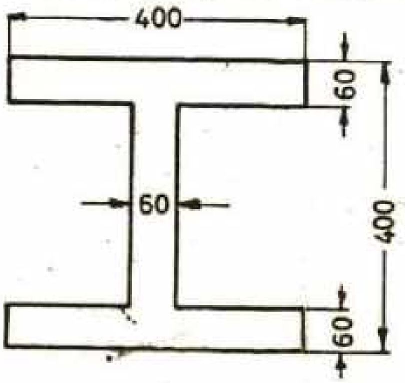
a)



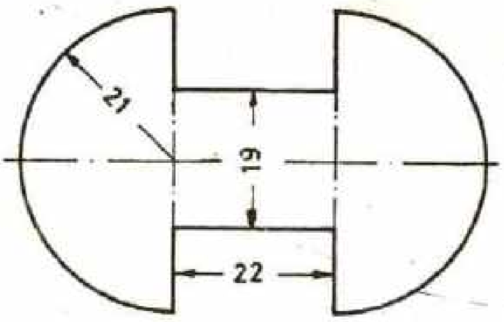
b)



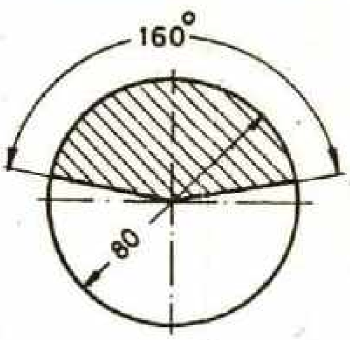
c)



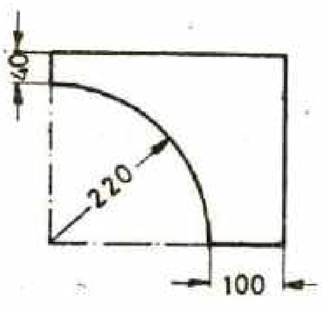
d)



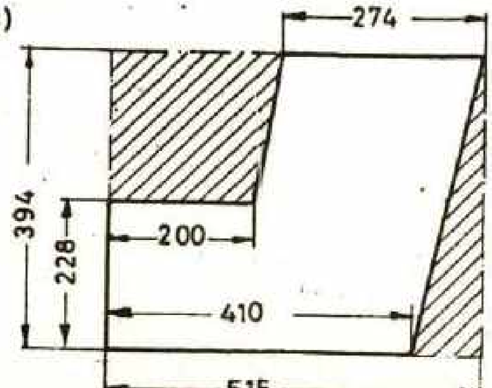
e)



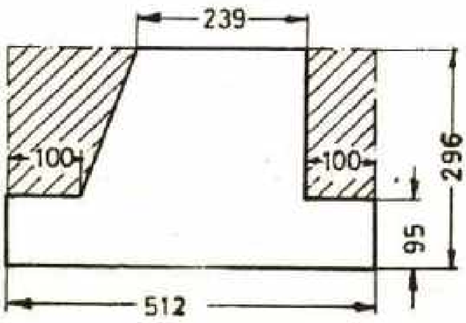
f)



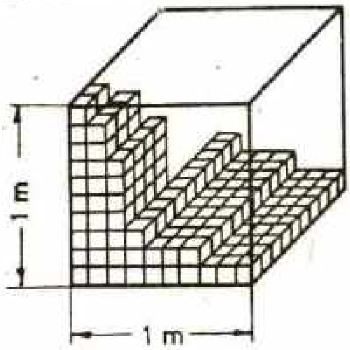
g)



h)



CHAPTER 11 VOLUMES



The formula for calculating the volume of engineering solids can be deduced both arithmetically and by means of drawings.

If we draw an engineering solid, e.g. a cube, on a squared paper, then the volume of the cube is represented by the number of cubes enclosed by it.

$V = 1\ m^3$

11.1 Units of Volume

The international metric units for measuring volumes are derived from the linear units in the same way as the units for areas, i.e. millimetre, centimetre, decimetre and metre:

$$1\ m^3 = 1\ m\ (length) \times 1\ m\ (depth) \times 1\ m\ (width)$$

Table:

$1\ m^3 = 1000\ dm^3$

$1\ dm^3 = 1000\ cm^3$

$1\ cm^3 = 1000\ mm^3$

Note: The factor for conversion from one unit to the next is 1000.

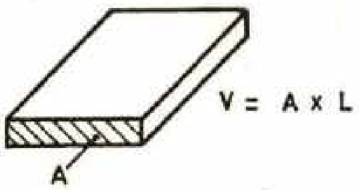
Remark: The previously used British units, e.g. cubic inch, cubic foot, have been replaced by the metric units

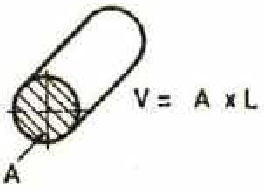
$$1\ cubic\ inch = 16.387\ cm^3$$

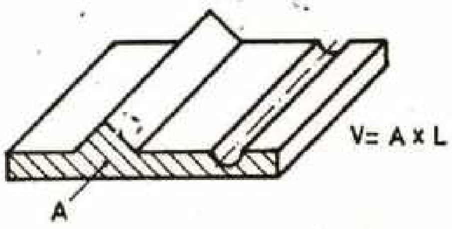
11.2 Volume of Parallel Engineering Solids

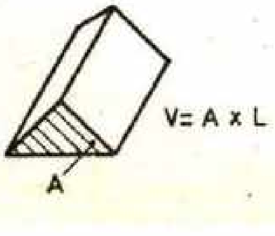
A parallel engineering solid has equal ground and top surface areas which are lying parallel to each other.

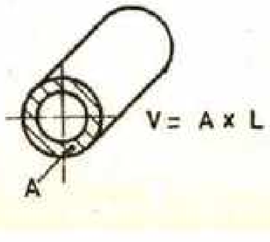
As a general rule the volume of these solids will be calculated by multiplying the surface area (cross section) by the length of the workpiece. This is to be done with both, simple and composed areas.

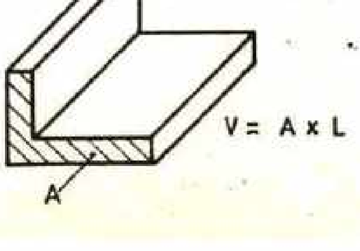




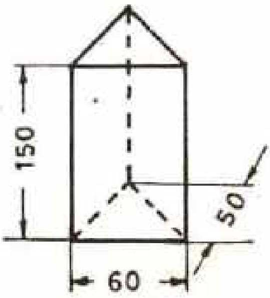








Example a)



Given: a = 60 mm; h = 50 mm; L = 150 mm;

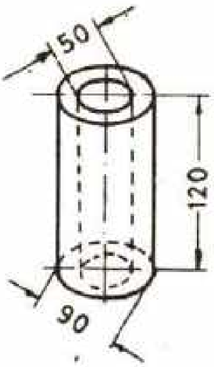
Solution: $V = A \times L$

$$A = \frac{a x h}{2} = \frac{6x5}{2} = \underline{15\ cm^2}$$

$$V = 15\ cm^2 \times 15\ cm$$

$$= \underline{\underline{225\ cm^3}}$$

Example b)



Given: D = 90 mm; d = 50 mm; L = 120 mm;

Solution: $V = A \times L$

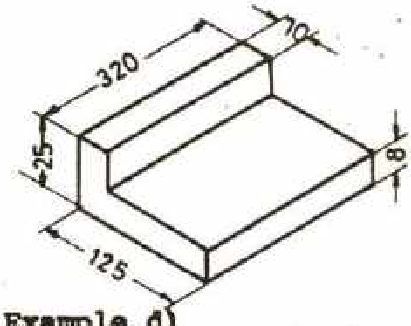
$$V = \left(\frac{D^2 x \pi}{4} - \frac{d^2 x \pi}{4}\right) x L$$

$$= \left(\frac{9^2 x \pi}{4} - \frac{5^2 x \pi}{4}\right) x 12$$

$$= (63.6\ cm^2 - 19.6\ cm^2) x 12\ cm$$

$$= 44\ cm^2 \times 12\ cm = \underline{\underline{528\ cm^3}}$$

Example c)



Solution: $V = A \times L$

$$A = A_1 + A_2$$

$$A_1 = axb = 10x25 = \underline{250\ mm^2}$$

$$A_2 = axb = 115x8 = \underline{920\ mm^2}$$

$$A = 250 + 920 = 1170\ mm^2$$

$$V = 1170\ mm^2 \times 320\ mm$$

$$= 374\ 400\ mm^3 = \underline{\underline{374.4\ cm^3}}$$

Example d)

$$V = A \times L$$

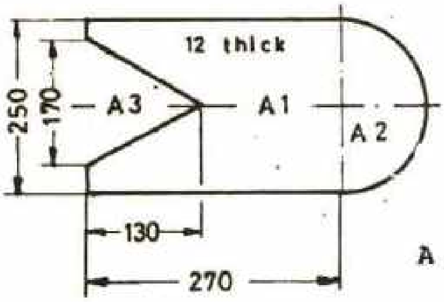
$$A = A_1 + A_2 - A_3$$

$$A_1 = a \times b = 27 \times 25 = 675\ cm^2$$

$$A_2 = \frac{d^2 x \pi}{4} : 2 = \frac{25^2 x 3.14}{4} : 2$$

$$= 491 : 2 = 245.5\ cm^2$$

$$A_3 = \frac{a \times h}{2} = \frac{17 \times 13}{2} = 110.5\ cm^2$$



$$A = 675\ cm^2 + 245.5\ cm^2 - 110.5\ cm^2$$

$$= \underline{810\ cm^2}$$

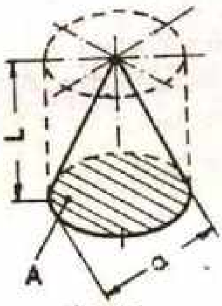
$$V = 810\ cm^2 x\ 1.2\ cm$$

$$= \underline{\underline{972\ cm^3}}$$

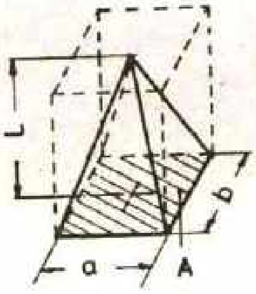
11.3 Volumes of Cones and Pyramids

In conical or pyramidical engineering solids the lines from the base-surface area are running to one point (cone point, pyramid point).

As a general rule the volume of these solids consists of one third the volume of parallel engineering solids with the same ground surface area and length.



CONE

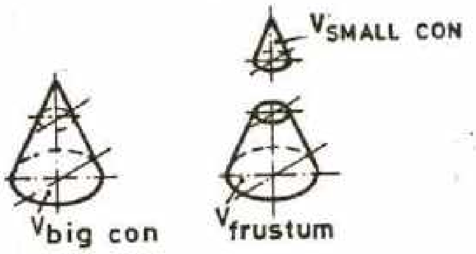


PYRAMID

When calculating the volume of cones or pyramids we, therefore, first calculate the volume of the parallel solid and, then, divide it by three.

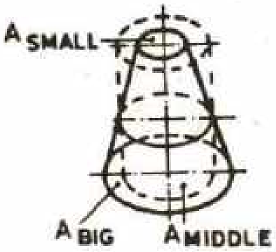
Formula: $\boxed{V = \frac{A \times L}{3}}$

11.4 Volumes of Frustums of Cones and Pyramids



Frustums are parts of cones or pyramids.
Their volume can be calculated by subtracting the volume of the cut-away (dotted) cone or pyramid from the complete unit.

$$\text{Volume}_{frustum} = V_{big\ cone} - V_{small\ cone}$$



Formula: $\boxed{V_f = V_C - V_c}$

The volume of a frustum can also be found by middling the two surfaces and then multiplying by the length, thus converting (theoretically) a frustum of a cone into a cylinder and the frustum of a pyramid into a prism.

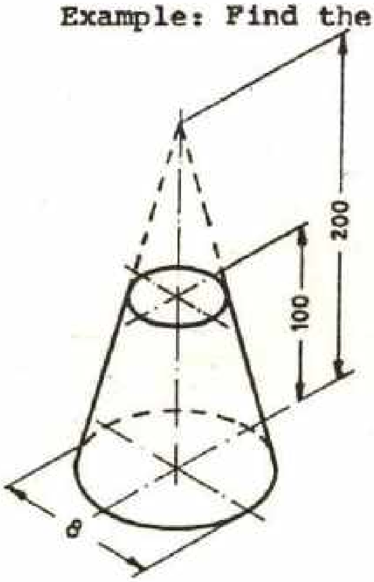
$$A_m = \frac{d_m^2 \times \pi}{4} \qquad \text{Volume}_{frustum} = A_{middled} \times \text{Length}$$

$$d_m = \frac{D + d}{2}$$

Formula: $\boxed{V_f = A_m \times L}$

Note: This method leads only to rough results. If there is a high accuracy required, this method must not be used.

Example: Find the volume using two methods.



Given: D = 80 mm; d = 40 mm; L_1 = 100 mm;
L_2 = 200 mm;

Solution a:

$$V = \frac{A_1 x L_1}{3} - \frac{A_2 x L_2}{3}$$

$$A_1 = \frac{D^2 x \pi}{4} = \frac{8^2 x \pi}{4}$$

$$= 50.3\ cm^2$$

$$A_2 = \frac{d^2 x \pi}{4} = \frac{4^2 x \pi}{4}$$

$$= 12.6\ cm^2$$

$$V = \frac{50.3x20}{3} - \frac{12.6x10}{3}$$

$$= 334.7\ cm^3 - 42.0\ cm^3$$

$$= \underline{\underline{292.7\ cm^3}}$$

Solution b:

$$V = A_m x\ L_2$$

$$A_m = \frac{d_m^2\ x \pi}{4}$$

$$d_m = \frac{D + d}{2} = 6\ cm$$

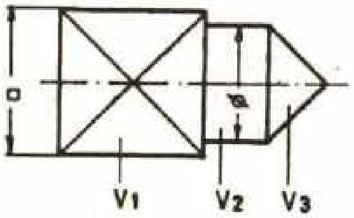
$$A_m = \frac{6^2\ x \pi}{4} = 28.3\ cm^2$$

$$V = 28.3\ cm^2 x\ 10\ cm$$

$$= \underline{\underline{283\ cm^3}}$$

// Compare the results!

11.5 Volumes of Composed Workpieces

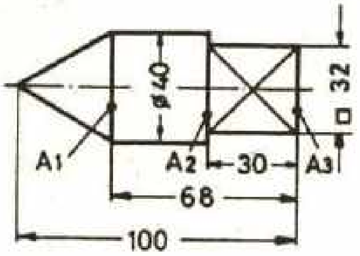


The volume of composed workpieces can be calculated simply by adding the volumes of the single parts together.

Formula: $\boxed{V = V_1 + V_2 + V_3 \ ......}$

Example: Calculate the volume of the centre point (cm^3):

Solution:



Volume 1: Cone
Volume 2: Cylinder
Volume 3: Prism

$$V = V_1 + V_2 + V_3$$

$$V_1 = \frac{A_1\ x\ L_1}{3};\quad A_1 = \frac{d^2 x \pi}{4} = \frac{4^2 x \pi}{4} = 12.6\ cm^2$$

$$V_1 = \frac{12.6\ cm^2 x\ 3.2\ cm}{3} = \underline{13.5\ cm^3}$$

$$V_2 = A_2\ x\ L_2\ ;\ A_2 = \frac{4^2\ x\ \pi}{4} = 12.6\ cm^2$$

$$V_2 = 12.6\ cm^2\ x\ 3.8\ cm = \underline{47.9\ cm^3}$$

$$V_3 = A_3\ x\ L_3 = (3.2 cm)^2 x\ 3\ cm = \underline{30.7\ c}$$

$$V = 13.5\ cm^3 + 47.9\ cm^3 + 30.7\ cm^3$$

$$= \underline{\underline{92.1\ cm^3}}$$

11.6 Quantities of Liquids

Generally the quantity of liquids is given in litres or parts thereof.
There is a close relation between the units for measuring the quantities of solids and those for measuring the quantities of liquids.

Table

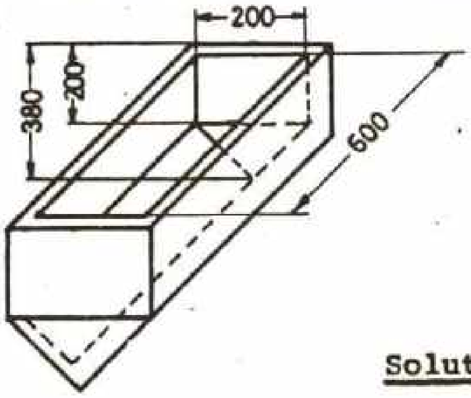
$1\ m^3 = 1000$ litres
$1\ dm^3 =$ 1 litre (l)
$1\ cm^3 =$ 1 millilitre (ml)

1 litre $= 1\ dm^3$
$= 1000\ cm^3$
1 millilitre $= 1\ cm^3$

Remark: The previously used British units, e.g. gallon, quart and pint have been replaced by the metric units.

1 gallon = 4.56 litres

To calculate the volume of a liquid stored in a container, we first calculate the inner volume of the container in $cm^3/dm^3/m^3$ and then convert into litre / millilitre.



Example:

Calculate the quantity of oil which can be stored in the container (in litres).

Solution:

$V = V_1 + V_2$

$V_1 = A_1 \times L$

$A_1 = a \times b = 20 \times 20 = 400\ cm^2$

$V_1 = 400\ cm^2 \times 60\ cm = \underline{24\ 000\ cm^3}$

$V_2 = A_2 \times L$

$A_2 = \frac{a \times h}{2} = \frac{20 \times 18}{2} = 180\ cm^2$

$V_2 = 180\ cm^2 \times 60\ cm = \underline{10\ 800\ cm^3}$

$V = 24{,}000\ cm^3 + 10{,}800\ cm^3 = 34{,}800\ cm^3$

$= 34.8\ dm^3 = \underline{\underline{34.8\ \text{litre}}}$

Exercises:

1) Convert in cm^3: 33480 mm^3; 0.166 dm^3; 165 mm^3.

2) Convert in litre: 14.5 dm^3; 15880 cm^3; 4800 ml.

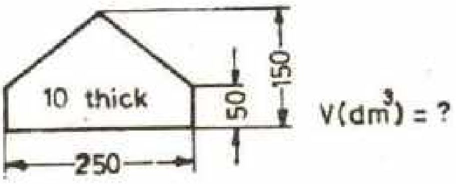
3) Add (in cm^3): a) 645 mm^3 + 0.84 dm^3 + 310 cm^3

b) 3870 mm^3 + 914 cm^3 - 0.012 dm^3

4) From a squared steel bar, 40 x 40, we cut off a length of 310 mm. Calculate the volume.

5) From a round bar, Ø 60, a job of 200 mm length is to be cut. Calculate its volume.

6)



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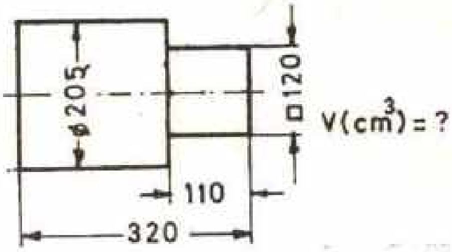

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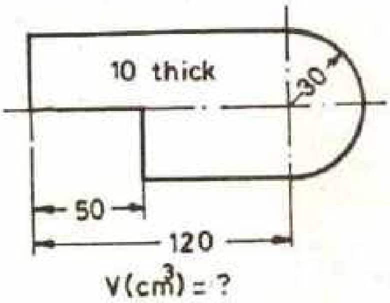

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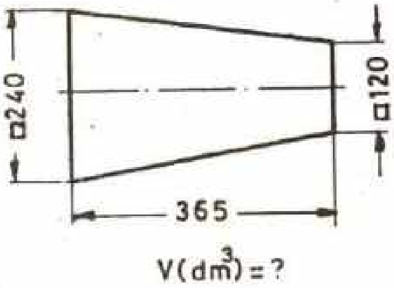
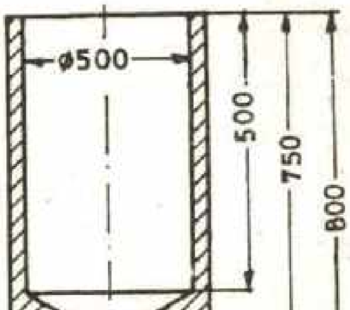


10) Problems:

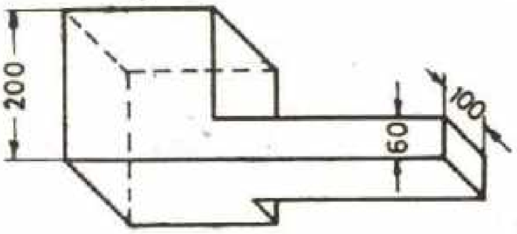
a) A cylindrical measuring glass contains 300 ml of oil. Its dia is 80 mm. Calculate its height.

b)



Calculate the amount of fluid which can be filled in (litre).

c)



A cubed piece of steel, a = 200 mm, will be forged to a steel bar, 60 x 100. Calculate the length of the rectangular bar !

CHAPTER 12 WEIGHT

The weight of material (solid, liquid or gasoid) is the force acting on the material due to the gravitational force of the earth.

When lifting a workpiece, we actually are applying a counter-force to the gravitational force of the earth.

The force which we use to lift a body is called the weight. It is expressed in Newton (N).

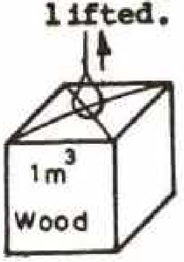
The amount of material of a body is called its mass. It is expressed in Kilogram (kg).

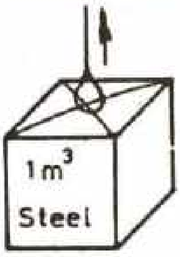
The mass of 1 kg has a weight of about 10 N.

Since in practical life people do not differentiate between mass and weight yet the same practice will be followed in this book.

12.1 Specific Gravity

Because of the different density of various materials, the force which is required to lift a certain body varies not only by its volume but also by the type of material to be lifted.





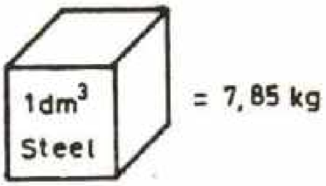
If we are lifting the same volume of different materials we can state that their weight is different. This is so because of their different density and consequently because of their different specific gravity.

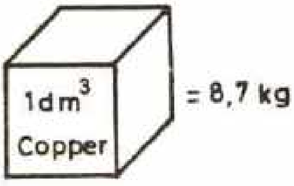
Generally the value of the specific gravity for a certain material is given without denomination. It is understood that for volumes expressed in

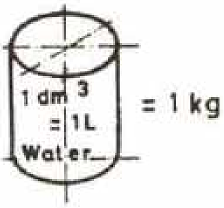
cm^3 or ml spec. gravity = gram per cm^3 / ml

dm^3 or 1 spec. gravity = kilogram per dm^3 / 1

m^3 spec. gravity = metric ton per m^3







<u>Table for spec. gravity of various materials</u>

Mild steel :	7.85	Water :	1.0
Cast iron :	7.5	Lubricating oil :	0.9
Aluminium :	2.7		
Copper :	8.7		
Brass :	8.5		
Zinc :	7.1		

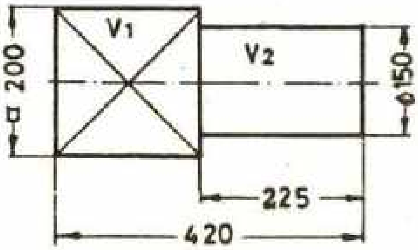
Note: Further specifications may be looked up in a table book.

12.2 Calculating Weight on the Basis of Volume

The weight of material can be calculated by multiplying the volume by the specific gravity, i.e. we first have to find the volume of a workpiece or a liquid, then multiply it by the unit volume of the material concerned.

weight	=	volume	x	spec. gravity
w	=	V	x	γ

Example a) Calculate the weight of the steel pin in kg.



Solution:

$w = V \times \gamma$

$V = V_1 + V_2$

$V_1 = A_1 \times h_1$

$A_1 = 400\ cm^2$

$V_1 = 400\ cm^2 \times 19.5\ cm$

$= \underline{7800\ cm^3}$

$V_2 = A_2 \times h_2$

$A_2 = 177\ cm^2$

$V_2 = 177\ cm^2 \times 22.5\ cm$

$= \underline{3982\ cm^3}$

$V = 7800\ cm^3 + 3982\ cm^3$

$= 11782\ cm^3 = \underline{11.78\ dm^3}$

$w = 11.78\ dm^3 \times 7.85\ kg/dm^3$

$= \underline{\underline{92.47\ kg}}$

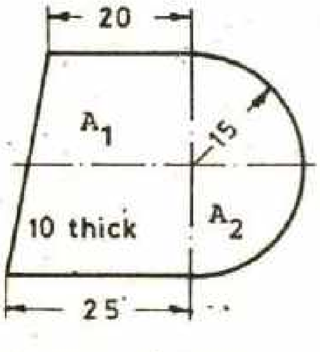
$A_1 = a \times a = 20 \times 20$

$= \underline{400\ cm^2}$

$$A_2 = \frac{d^2\pi}{4} = \frac{15^2 \times \pi}{4}$$

$= \underline{177\ cm^2}$

Example b) Calculate the weight of 200 brass covers



Solution:

$w = V \times \gamma$

$V = A \times h$

$A = A_1 + A_2$

$A_1 = 6.75\ cm^2$

$A_2 = 3.53\ cm^2$

$V = 10.28\ cm^2 \times 1.0\ cm = \underline{10.28\ cm^3}$

$w = 10.28\ cm^3 \times 8.7\ g/cm^3 = \underline{89.436\ g}$

Total (200 pieces) = 89.436 g x 200

= 17887.2 g

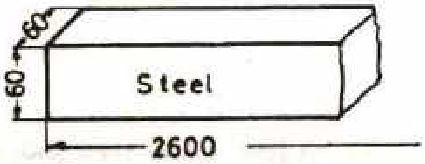
= $\underline{\underline{17.887\ kg}}$

12.3 Calculating Weight on the Basis of Length

When buying steel bars, such as square, round, hexagonal, rectangular, angle iron, U-iron or T-iron, the weight of the material often is calculated on the basis of length. That means, the weight of one unit of length is known. The total weight of the material in this case is calculated by multiplying the total length by the weight of the unit length of the material:

$$\boxed{w_{total} = w_{unit} \times L_{total}}$$

Example: Calculate the weight of a square steel bar, 60 x 60, 2600 mm length.



1 m = 28.3 kg
(from table next page)

Given: $w_{unit} = 28.3$ kg/m
$L_{total} = 2600$ mm $= 2.6$ m

Solution:

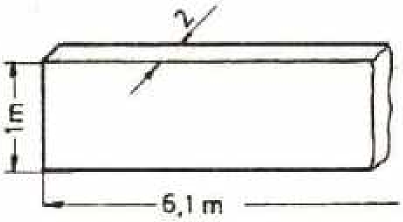
$$w_t = w_u \times L_t$$
$$= 28.3 \text{ kg/m} \times 2.6 \text{ m}$$
$$= \underline{73.58 \text{ kg}}$$

12.4 Calculating Weight on the Basis of Area

In the case of sheet steel the weight per unit is not based on the length but on the area of the material. The weight per unit area is:

$$w_{total} = w_{unit} \times A_{total}$$

Example: Calculate the weight of a steel sheet of 6.1 m^2; thickness: 3 mm



1 m^2 = 23.55 kg
(from table)

Given: $w_{unit} = 23.55$ kg/m^2
$A_{total} = 6.1$ m^2

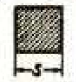
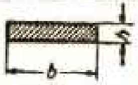
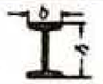
Solution:

$$w_t = w_u \times A_t$$
$$= 23.55 \text{ kg/m}^2 \times 6.1 \text{ m}^2$$
$$= 144.6 \text{ kg}$$

========

Exercises:

1) Use the table for calculating the weight as given under nos. a – e

S															
mm	kg/m	kg/m	kg/m	b×h	kg/m	d	kg/m^2	b×b×d	kg/m	Bez.	h×b	kg/m	Bez.	h×b	kg/m
5	0,196	0,154	0,170	10 x 5	0,39	1	7,85	15 x 15 x 3	0,64	30	30 x 15	1,74	80	80 x 42	5,95
6	0,283	0,222	0,245	10 x 8	0,63	2	15,7	20 x 20 x 4	1,14	40	40 x 20	2,75	100	100 x 50	8,32
7	0,385	0,302	0,333	12 x 5	0,47	3	23,55	25 x 25 x 4	1,45	50	50 x 25	4,32	120	120 x 58	11,2
8	0,502	0,395	0,435	15 x 5	0,59	4	31,4	30 x 30 x 3	1,36	60	60 x 30	5,07	140	140 x 66	14,4
9	0,636	0,499	0,551	15 x 10	1,18	5	39,25	30 x 30 x 5	2,18	65	65 x 42	7,09	160	160 x 74	17,9
10	0,785	0,617	0,680	20 x 5	0,78	6	47,10	35 x 35 x 4	2,1	80	80 x 45	8,64	180	180 x 82	21,9
12	1,13	0,888	0,980	20 x 10	1,57	7	56,3	35 x 35 x 6	3,04	100	100 x 50	10,6	200	200 x 90	26,3
14	1,54	1,21	1,33	25 x 5	0,98	8	62,8	40 x 40 x 4	2,42	120	120 x 55	13,4	220	220 x 98	31,1
16	2,01	1,58	1,74	25 x 15	2,94	9	72	40 x 40 x 6	3,52	140	140 x 60	16,0	240	240 x 106	36,2
18	2,54	2,00	2,20	30 x 5	1,18	10	78,5	45 x 45 x 5	3,38	160	160 x 65	18,8	260	260 x 113	41,9
20	3,14	2,47	2,72	35 x 5	1,37	12	96	45 x 45 x 7	4,60	180	180 x 70	22,0	280	280 x 119	48,0
24	4,52	3,55	3,92	40 x 10	3,14	14	112	50 x 50 x 5	3,77	200	200 x 75	25,3	300	300 x 125	54,2
28	6,15	4,83	5,33	40 x 25	7,85	15	120	50 x 50 x 9	6,47	240	240 x 85	33,2	320	320 x 131	61,1
30	7,07	5,55	6,12	45 x 30	10,6	16	125	55 x 55 x 6	4,95	280	280 x 95	41,8	340	340 x 137	68,1
40	12,6	9,87	10,88	50 x 20	7,85	20	160	60 x 60 x 6	5,42	300	300 x 100	46,2	360	360 x 143	76,2
50	19,63	15,41	17,00	50 x 40	15,7	25	196	65 x 65 x 7	6,83	350	350 x 100	60,6	380	380 x 149	84,0
60	28,3	22,2	24,5	60 x 20	9,42	30	240	70 x 70 x 7	7,38	400	400 x 110	71,8	400	400 x 155	92,6
70	38,5	30,2	33,3	70 x 30	16,5	40	320	75 x 75 x 7	7,94	450	450 x 170	115	450	450 x 170	115
80	50,2	39,5	43,5	80 x 40	25,1	50	392	80 x 80 x 8	9,66	500	500 x 185	141	500	500 x 185	141
90	63,6	49,9	55,1	90 x 50	35,3	60	480	90 x 90 x 9	12,2	550	550 x 200	167	550	550 x 200	167

a) Calculate the weight of a square steel bar □ 24 x 1500.

b) Calculate the weight of a round steel bar Ø 12 x 2400.

c) Calculate the weight of a sheet steel 200 x 900, 6 thick.

d) Calculate the total weight of three pieces of angle iron L 80 x 80 x 8 x 180

e) For producing a railing the following items are required:
2 pieces L 30 x 30 x 5 x 1250
2 pieces L 30 x 30 x 3 x 2300
2 pieces ▭ 12 x 5 x 1240
5 pieces Ø 6 x 2300

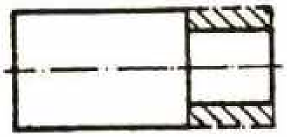
Calculate the total weight of the railing.

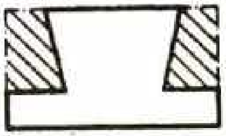
2) Calculate the weight of exercise no. 7 and 9 of chapter 11. Material: Aluminium. γ = 2.6 kg/dm^3

3) Find the total weight if 5 pieces of U-steel bar, 150 mm long, are required

CHAPTER 13 ESTIMATION AND WASTAGE

When machining or cutting a job we generally have to consider a certain loss of material, the wastage.





The wastage is the difference between the estimated material (taken from the store) and the material of the finished job.

13.1 Estimation

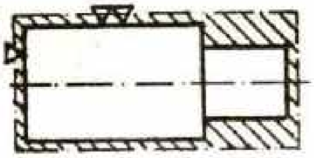
The estimated material is the material required for a certain job.
For estimation the following details are to be considered:

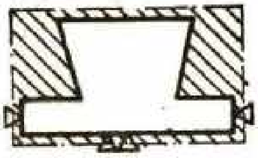
a) The amount of wastage caused by the shape of the job.

b) The amount of wastage caused by the required accuracy and surface finish of the job.

c) The amount of wastage caused by the non-availability of the raw material with the required dimensions.

In order to manufacture a workpiece with the shape, dimensions and surface quality required we must, therefore, select the suitable raw material by considering not only the outer dimensions of the finished workpiece but also the extra material for machining and cutting.





Note: To save valuable material only necessary extra material is to be added.

13.2 Wastage

The wastage can be calculated either by subtracting the volume of the finished workpiece from the volume of the estimated material or by subtracting the weights from each other accordingly.

Formulae: Wastage = Volume (estimated) - Volume (finished)

or = weight (estimated) - weight (finished)

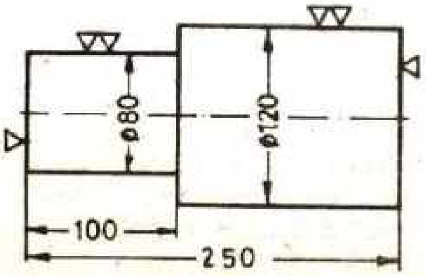
Since wastage is generally expressed in per cent, the second step would be to find this value.

Estimate = 100 % Wastage = ? %

Note: The estimated material is always to be taken as 100 % !

Calculating the Wastage on the Basis of Volume

Example: Calculate the wastage in % for machining the bolt. A round bar, ∅ 130 x 260 is available.



Given: $d_e = 130$ mm, $l_e = 260$ mm · drawing

Solution:

$$\text{Wastage} = V_{estimate} - V_{finish}$$
(cm^3)

$$V_e = A_e \times h_e$$
$$= 132.7\ cm^2 \times 26\ cm$$
$$= \underline{3450.2\ cm^2}$$

$$V_f = V_1 + V_2$$
$$V_1 = A_1 x h_1 = 50.24 cm^2 x 10 cm$$
$$= 502.4\ cm^3$$
$$V_2 = A_2 x h_2 = 113.04 cm^2 x 15 cm$$
$$= 1695.6\ cm^3$$

$$V_f = 502.4\ cm^3 + 1695.6\ cm^3$$
$$= \underline{2198.0\ cm^3}$$

$$\text{Wastage} = 3450.2\ cm^3 - 2198.0\ cm^3$$
$$= 1252.2\ cm^3$$

Wastage in %:

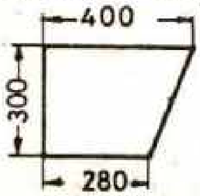
$3450.2\ cm^3 = 100$ %

$$1251\ cm^3 = \frac{100 x 1251}{3450.2}\ \%$$
$$= 36.2\ \%$$

Calculating the Wastage on the Basis of Area and Length

Example a)

Calculate the wastage in % when a sheet of 300 x 400 is drawn from the store.



$$\text{Wastage} = A_{estim.} - A_{finish}$$
(cm^2)

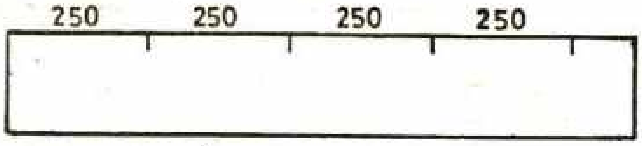
$$A_e = 30\ cm \times 40\ cm$$
$$= 1200\ cm^2$$
$$A_f = \frac{40+28}{2} x 30 = \frac{68}{2} x 30$$
$$A_f = 1020\ cm^2$$

$$\text{Wastage} = 1200\ cm^2 - 1020\ cm^2$$
(cm^2)
$$= 180\ cm^2$$

$$\text{Wastage} = \frac{180 \times 100\ \%}{1200} = 15\ \%$$
(%)

Example b)

Calculate the wastage in % when a flat steel bar 20 x 6 x 1200 is to be cut.



$$\text{Wastage} = L_{estim.} - L_{finish}$$
(mm)

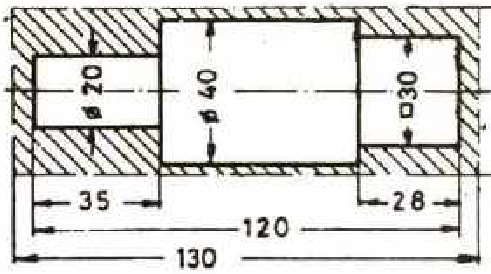
$$L_e = 1200\ mm$$
$$L_f = 4 \times 250\ mm$$
$$= 1000\ mm$$

$$\text{Wastage} = 1200 - 1000 = 200\ mm$$
(mm)

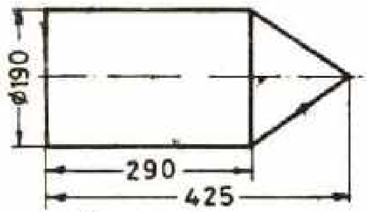
$$\text{Wastage} = \frac{200\ cm \times 100\ \%}{1200\ cm}$$
$$= 16.66\ \%$$

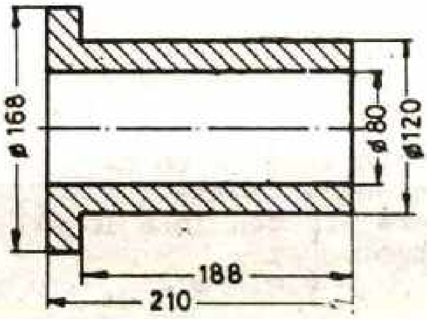
Exercises:

1) Calculate the wastage in cm^3 and per cent.



2) Calculate the wastage in cm^3 and percent when a steel bar, dia 200 mm and 440 mm length is available.





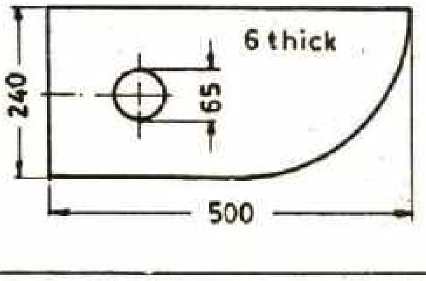
3) Calculate the wastage in per cent,

a) if the bronce bushing is turned from a solid bronce bar, Ø 175 mm and a length of 215 mm.

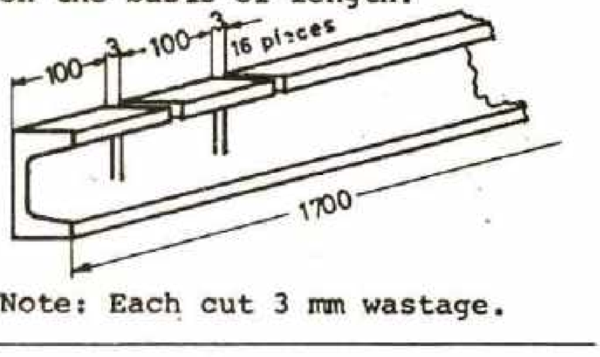
b) if the bushing is to be turned from a hollow cast bronce bar. Estimate material: All dimensions plus 5 mm machining allowance.

c) Compare the results, particularly with the view that mass production is planned.

4) Calculate the wastage in % on the basis of area.

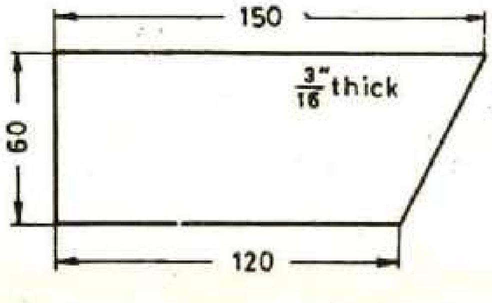


5) Calculate the wastage in % on the basis of length.



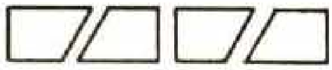
Note: Each cut 3 mm wastage.

6) Out of a metal sheet, 3 feet x 6 feet, 3/16" thick, as many cover sheets as possible are to be sheared.



a) Calculate the number of cover sheets possible.

b) Calculate the wastage in %.

Note:  etc !

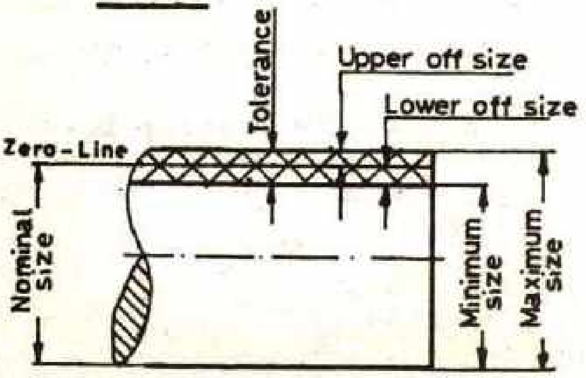
CHAPTER 14 CALCULATION OF FITS

The accuracy of workpieces can be planned depending upon the requirements such as changeability and production costs.

In this connection a tolerance fixes the range of accuracy for one workpiece (see chapter 6) only.
If two workpieces are working together (e.g. shaft and pulley) the tolerance of each part must be designed in a way that they exactly serve the desired purpose (play or interference between shaft and pulley).
In this case we have a fit.

14.1 Basic Definitions

Shaft:

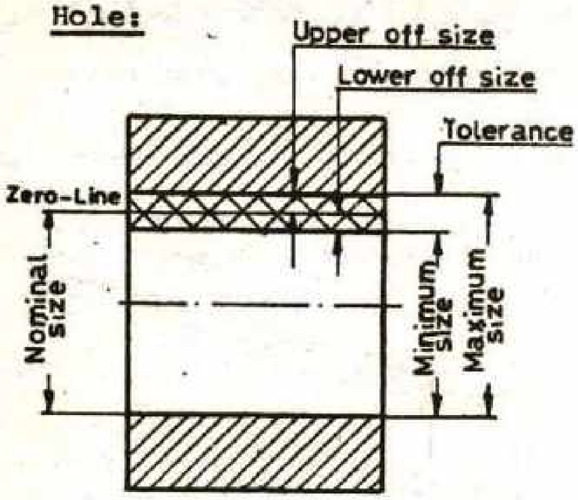


All external features of a part including parts which are not cylindrical.

The tolerance is to be expressed with small letters written in a lower position,

e.g. Ø 25_{g6}

Hole:



All internal features of a part including parts which are not cylindrical.

The tolerance is to be expressed with capital letters written in the upper position,

e.g. Ø 25^{H7}

Zero-Line: In a graphical representation of off-sizes and fits the straight line to which the deviations are referred to is called the zero-line.

Off-sizes are to be calculated from the zero-line to upper and lower off-sizes.

14.2 Meaning of symbols

ISO-fits are always expressed in symbols with the following meaning:

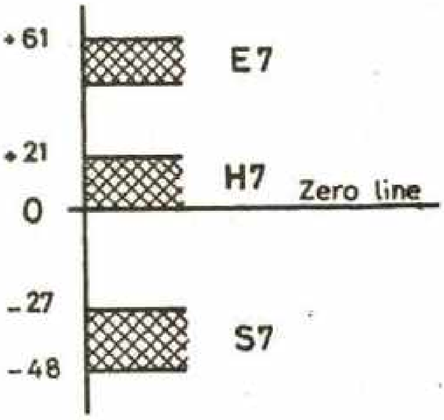
Letters: They indicate the position of the tolerance field, e.g.

$\varnothing 25^{E7} = \varnothing 25^{+0.061}_{+0.040}$

$\varnothing 25^{H7} = \varnothing 25^{+0.021}_{-0}$

$\varnothing 25^{S7} = \varnothing 25^{-0.027}_{-0.048}$

Capital letters refer to the hole, small letters refer to the shaft.

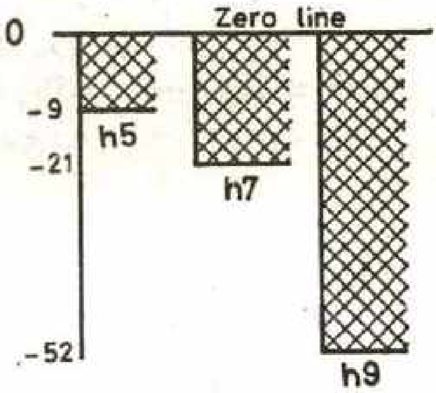


Figures: They indicate the size of the tolerance field, e.g.

$\varnothing 25_{h5} = \varnothing 25^{+0}_{-0.009}$

$\varnothing 25_{h7} = \varnothing 25^{+0}_{-0.021}$

$\varnothing 25_{h9} = \varnothing 25^{+0}_{-0.052}$



14.3 Calculating the Fit-Dimensions

We have to consult a table (shown below is an extract).

ISO FITS (Extract)

	Deviations in µm(=0,001mm)					
Nominal sizes/mm	Hole	Shafts				
over...to	H7	r 6	n 6	j6	e8	d9
10 ····· 14	+18 0	+34 +23	+23 +12	+8 -3	-32 -59	-50 -93
14 ····· 18						
18 ····· 24	+21 0	+41 +28	+28 +15	+9 -4	-40 -73	-65 -117
24 ····· 30						
30 ····· 40	+25 0	+50 +34	+33 +17	+11 -5	-50 -89	-80 -142
40 ····· 50						
50 ····· 65	+30 0	+60 +41	+39 +20	+12 -7	-60 -106	-100 -174
65 ····· 80		+62 +43				
80 ····· 100	+35 0	+73 +51	+45 +23	+13 -9	-72 -126	-120 -207
100 ····· 120		+76 +54				

Example a) $\varnothing 35^{H7}$

Nominal size:	35 mm
Upper off-size:	+ 25 m =+0.025 mm
Lower off-size:	0
Maximum size:	35.025 mm
Minimum size:	35.000 mm

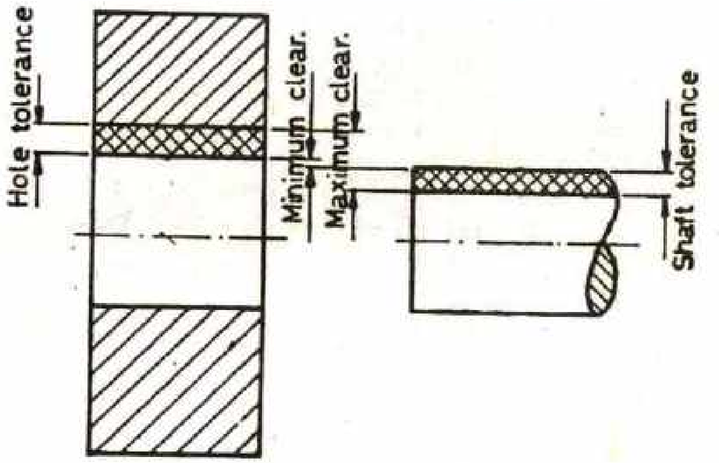
Example b) $\varnothing 95^{H7}_{r6}$

Nominal size:	95 mm
Hole	
Upper off-size:	+ 35 m = +0.035 mm
Lower off-size:	0
Maximum size:	95.035 mm
Minimum size:	95.000 mm
Shaft	
Upper off-size:	+ 73 m = +0.073 mm
Lower off-size:	+ 51 m = +0.051 mm
Maximum size:	95.073 mm
Minimum size:	95.051 mm

Note: Further specifications may be looked up in a table book.

14.4 Clearance

The difference between the sizes of shaft and hole, provided the size of the hole is bigger than the size of the shaft, is called the clearance.



Maximum clearance =
maximum size hole -
minimum size shaft

Minimum clearance =
minimum size hole -
maximum size shaft

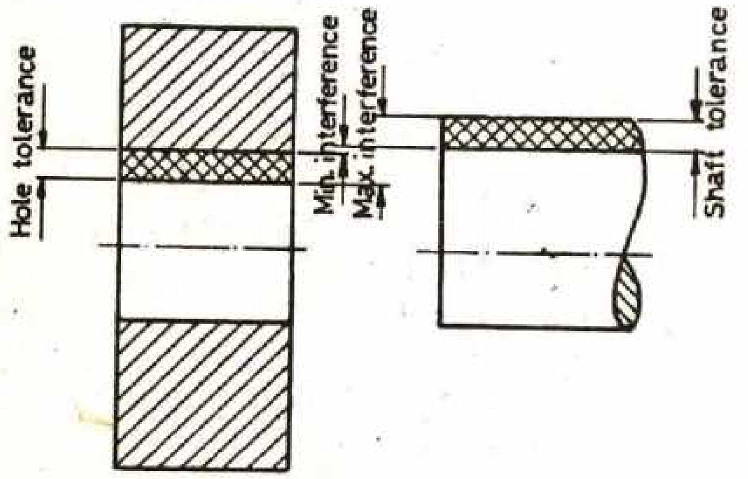
Example: Ø 40^{H7}_{e8}

Maximum clearance = max. size hole - min. size shaft
= 40.025 mm - 39.911 mm = <u>0.114 mm</u>

Minimum clearance = min. size hole - max. size shaft
= 40.000 mm - 39.950 mm = <u>0.050 mm</u>

14.5 Interference

The difference between the sizes of shaft and hole, provided the size of the shaft is bigger than the size of the hole, is called interference.



Maximum interference =
maximum size shaft -
minimum size hole

Minimum interference =
minimum size shaft -
maximum size hole

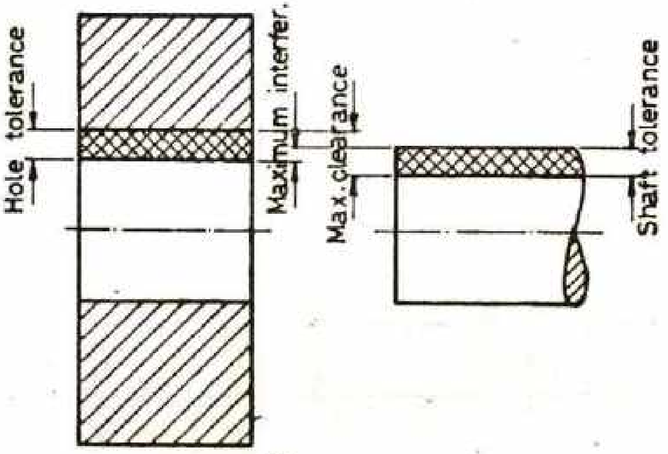
Example: Ø 85^{H7}_{r6}

Maximum interference = max. size shaft - min. size hole
= 85.073 mm - 85.000 mm = <u>0.073 mm</u>

Minimum interference = min. size shaft - max. size hole
= 85.051 mm - 85.035 mm = <u>0.016 mm</u>

14.6 Transition Fit

In a transition fit the tolerances of shaft and hole are overlapping. This may result in clearance or interference. There is no minimum clearance and no minimum interference.



Maximum clearance =
max. size hole -
min. size shaft

Maximum interference =
max. size shaft -
min. size hole

Example: Ø 60^{H7}_{j6}

Maximum clearance = max. size hole - min. size shaft
= 60.030 mm - 59.993 mm = 0.037 mm

Maximum interference = max.size shaft - min. size hole
= 60.012 mm - 60.000 mm = 0.012 mm

Calculate the dimensions of the following fits and indicate if it is a clearance fit, an interference fit or a transition fit.

Example a) Ø 20^{H7}_{d9}

Solution:

Hole: Max. size = 20.021 mm   Shaft: Max. size = 19.935 mm
Min. size = 20.000 mm   Min. size = 19.883 mm

The size of the hole is always bigger than the size of the shaft. Therefore we have a clearance fit.

Example b) Ø 70^{H7}_{n6}

Solution:

Hole: Max. size = 70.030 mm   Shaft: Max. size = 70.039 mm
Min. size = 70.000 mm   Min. size = 70.020 mm

The size of the hole may be bigger than the size of the shaft.
Max. clearance = max. size hole - min. size shaft
= 70.030 mm - 70.020 mm = 0.010 mm
The size of the shaft may be bigger than the size of the hole:
Max. interference = max. size shaft - min. size hole
= 70.039 mm - 70.000 mm = 0.039 mm

Therefore we have a transition fit.

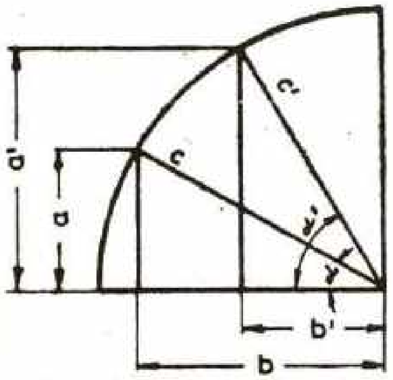
Exercises:

Supply the missing items in the table.
Use the table given under 14.3.

ISO Fit	HOLE Max. size Min. size	SHAFT Max. size Min. size	Clearance Max. Min.	Interference Max. Min.	Kind of Fit
$18\frac{H7}{r6}$	18.018	18.034	---	0.034	Inter-ference Fit
	18.000	18.023	---	0.005	
$30\frac{H7}{j6}$					
$90\frac{H7}{d9}$					
$120\frac{H7}{n6}$					
$50\frac{H7}{e8}$					
	40.025	40.033			
	40.000	40.017			
85^{H7}		84.880			
		84.793			
	25.021		---	0.041	
	25.000		---	0.007	
68^{H7}			0.136		
			0.060		
	100.035		0.044	0.013	
	100.000		---	---	

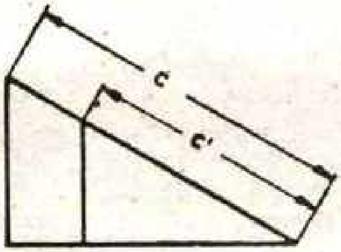
CHAPTER 15 TRIGONOMETRIC RATIOS

Trigonometric ratios express the relations between two sides of a right angled triangle and an angle of the triangle.



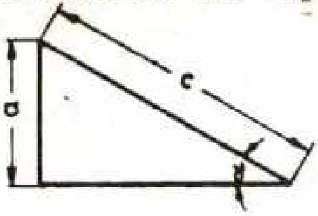
As shown in the diagram the length of side "a" (perpendicular) increases as the angle "α" increases.

Similarly, the length of side "b" (base) decreases when side "a" and angle "α" increase.



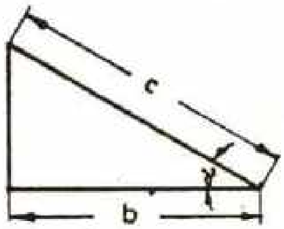
When keeping angle "α" but changing the length of one side, e.g. side "c" (hypotenuse), the other sides are also affected.

There are four possible relations amongst the sides which influence the size of the angle "α" for which we have special expressions.



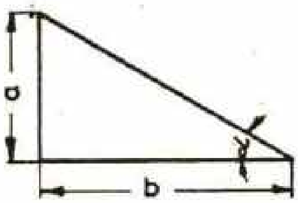
1) The relation between perpendicular and hypotenuse is called sine

$$\frac{\text{perpendicular}}{\text{hypotenuse}} = \text{sine} \quad \underline{\text{or}} \quad \boxed{\frac{a}{c} = \sin \alpha}$$



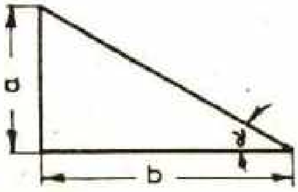
2) The relation between base and hypotenuse is called cosine

$$\frac{\text{base}}{\text{hypotenuse}} = \text{cosine} \quad \underline{\text{or}} \quad \boxed{\frac{b}{c} = \cos \alpha}$$



3) The relation between perpendicular and base is called tangent

$$\frac{\text{perpendicular}}{\text{base}} = \text{tangent} \quad \underline{\text{or}} \quad \boxed{\frac{a}{b} = \tan \alpha}$$



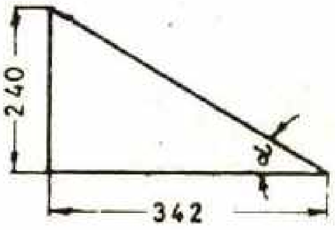
4) The relation between base and perpendicular is called cotangent

$$\frac{\text{base}}{\text{perpendicular}} = \text{cotangent} \quad \underline{\text{or}} \quad \boxed{\frac{b}{a} = \cot \alpha}$$

15.1 Reading Trigonometric Tables

A system has been worked out that gives us the exact value of an angle "α" if the value of its trigonometric ratio is known.

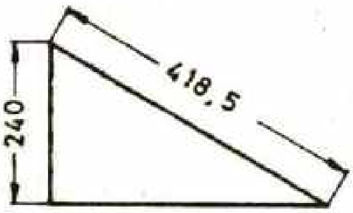
Example: Find the value of angle "α" in degrees, minutes and seconds.



$$\tan\alpha = \frac{\text{perpendicular}}{\text{base}} = \frac{240}{342}$$

$$= \underline{0.7002}$$

The tangent of 0.7002 must now be looked up in a table.



$$\sin\alpha = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{240}{418.5}$$

$$= \underline{0.5736}$$

Now the sine of 0.5736 must be looked up in the table.

Tangent-table (extract)

	Minutes			
Degree	0	10	20	30
20				
25				
30	0.5774	0.5812	0.5851	0.5890
35	0.7002	0.7046	0.7089	0.7177
40	0.8391			

Sine-table (extract)

	Minutes		
Degree	0	10	20
20			
25			
30	0.5000	0.5025	0.5050
35	0.5736	0.5760	0.5783
40	0.6428	0.6450	

Result

$35^\circ \longleftarrow 0.7002$

$\tan 0.7002 = 35^\circ$

In our triangle the angle α has 35 degrees.

Result

$35^\circ \longleftarrow 0.5736$

$\sin 0.5736 = 35^\circ$

In our triangle the angle α has 35 degrees.

Exercises:

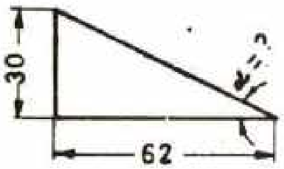
1) Find from the table the values of tan 0.0699; tan 0.3640; tan 0.3346; tan 0.5890 and tan 0.3607.

2) Find from the table the values of sin 0.1736; sin 0.3311.

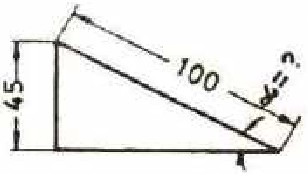
3) Find the tangent from the angles 15°; 20°30'; 42°40'.

4) Find the sine from the angles 20°; 25°50'; 2°10".

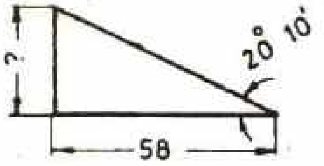
5)



Find the size of angle.



Find the size of angle.



Find the size of the perpendicular.

Tangent ∢ 0...45							
Degrees	Minutes						
	0	10	20	30	40	50	
0	0,0000	0,0029	0,0058	0,0087	0,0116	0,0145	0,0175
1	0,0175	0,0204	0,0233	0,0262	0,0291	0,0320	0,0349
2	0,0349	0,0378	0,0407	0,0437	0,0466	0,0495	0,0524
3	0,0524	0,0553	0,0582	0,0612	0,0641	0,0670	0,0699
4	0,0699	0,0729	0,0758	0,0787	0,0816	0,0846	0,0875
5	0,0875	0,0904	0,0934	0,0963	0,0992	0,1022	0,1051
6	0,1051	0,1080	0,1110	0,1139	0,1169	0,1198	0,1228
7	0,1228	0,1257	0,1287	0,1317	0,1346	0,1376	0,1405
8	0,1405	0,1435	0,1465	0,1495	0,1524	0,1554	0,1584
9	0,1584	0,1614	0,1644	0,1673	0,1703	0,1733	0,1763
10	0,1763	0,1793	0,1823	0,1853	0,1883	0,1914	0,1944
11	0,1944	0,1974	0,2004	0,2035	0,2065	0,2095	0,2126
12	0,2126	0,2156	0,2186	0,2217	0,2247	0,2778	0,2309
13	0,2309	0,2339	0,2370	0,2401	0,2432	0,2462	0,2493
14	0,2493	0,2524	0,2555	0,2586	0,2617	0,2648	0,2679
15	0,2679	0,2711	0,2742	0,2773	0,2805	0,2836	0,2867
16	0,2867	0,2899	0,2931	0,2962	0,2994	0,3026	0,3057
17	0,3057	0,3089	0,3121	0,3153	0,3185	0,3217	0,3249
18	0,3249	0,3281	0,3314	0,3346	0,3378	0,3411	0,3443
19	0,3443	0,3476	0,3508	0,3541	0,3574	0,3607	0,3640
20	0,3640	0,3673	0,3706	0,3739	0,3772	0,3805	0,3839
21	0,3839	0,3872	0,3906	0,3939	0,3973	0,4006	0,4040
22	0,4040	0,4074	0,4108	0,4142	0,4176	0,4210	0,4245
23	0,4245	0,4279	0,4314	0,4348	0,4383	0,4417	0,4452
24	0,4452	0,4487	0,4522	0,4557	0,4592	0,4628	0,4563
25	0,4663	0,4699	0,4734	0,4770	0,4806	0,4841	0,4877
26	0,4877	0,4913	0,4950	0,4986	0,5022	0,5059	0,5095
27	0,5095	0,5132	0,5169	0,5206	0,5243	0,5280	0,5317
28	0,5317	0,5354	0,5392	0,5430	0,5467	0,5505	0,5543
29	0,5543	0,5581	0,5619	0,5658	0,5696	0,5735	0,5774
30	0,5774	0,5812	0,5851	0,5890	0,5930	0,5969	0,6009
31	0,6009	0,6048	0,6088	0,6128	0,6168	0,6208	0,6249
32	0,6249	0,6289	0,6330	0,6371	0,6412	0,6453	0,6494
33	0,6494	0,6536	0,6577	0,6619	0,6661	0,6703	0,6745
34	0,6745	0,6787	0,6830	0,6873	0,6916	0,6959	0,7002
35	0,7002	0,7046	0,7089	0,7133	0,7177	0,7221	0,7265
36	0,7265	0,7310	0,7355	0,7400	0,7445	0,7490	0,7536
37	0,7536	0,7581	0,7627	0,7673	0,7720	0,7766	0,7813
38	0,7813	0,7850	0,7907	0,7954	0,8002	0,8050	0,8098
39	0,8098	0,8146	0,8195	0,8243	0,8292	0,8342	0,8391
40	0,8391	0,8441	0,8491	0,8541	0,8591	0,8642	0,8693
41	0,8693	0,8744	0,8796	0,8847	0,8899	0,8952	0,9004
42	0,9004	0,9057	0,9110	0,9163	0,9217	0,9271	0,9325
43	0,9325	0,9380	0,9435	0,9490	0,9545	0,9601	0,9657
44	0,9657	0,9713	0,9770	0,9827	0,9884	0,9942	1,0000

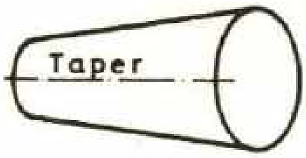
Sine ∢ 0...45							
Degrees	Minutes						
	0	10	20	30	40	50	
0	0,0000	0,0029	0,0058	0,0087	0,0116	0,0145	0,0175
1	0,0175	0,0204	0,0233	0,0262	0,0291	0,0320	0,0349
2	0,0349	0,0378	0,0407	0,0436	0,0465	0,0494	0,0523
3	0,0523	0,0552	0,0581	0,0610	0,0640	0,0669	0,0698
4	0,0698	0,0727	0,0756	0,0785	0,0814	0,0843	0,0872
5	0,0872	0,0901	0,0929	0,0958	0,0987	0,1016	0,1045
6	0,1045	0,1074	0,1103	0,1132	0,1161	0,1190	0,1219
7	0,1219	0,1248	0,1276	0,1305	0,1334	0,1363	0,1392
8	0,1392	0,1421	0,1449	0,1478	0,1507	0,1536	0,1564
9	0,1564	0,1593	0,1622	0,1650	0,1679	0,1708	0,1736
10	0,1736	0,1765	0,1794	0,1822	0,1851	0,1880	0,1908
11	0,1908	0,1937	0,1965	0,1994	0,2022	0,2051	0,2079
12	0,2079	0,2108	0,2136	0,2164	0,2193	0,2221	0,2250
13	0,2250	0,2278	0,2306	0,2334	0,2363	0,2391	0,2419
14	0,2419	0,2447	0,2476	0,2504	0,2532	0,2560	0,2588
15	0,2588	0,2616	0,2644	0,2672	0,2700	0,2728	0,2756
16	0,2756	0,2784	0,2812	0,2840	0,2868	0,2896	0,2924
17	0,2924	0,2952	0,2979	0,3007	0,3035	0,3062	0,3090
18	0,3090	0,3118	0,3145	0,3173	0,3201	0,3228	0,3256
19	0,3256	0,3283	0,3311	0,3338	0,3365	0,3393	0,3420
20	0,3420	0,3448	0,3475	0,3502	0,3529	0,3557	0,3584
21	0,3584	0,3611	0,3638	0,3665	0,3692	0,3719	0,3746
22	0,3746	0,3773	0,3800	0,3827	0,3854	0,3881	0,3907
23	0,3907	0,3934	0,3961	0,3987	0,4014	0,4041	0,4067
24	0,4067	0,4094	0,4120	0,4147	0,4173	0,4200	0,4226
25	0,4226	0,4253	0,4279	0,4305	0,4331	0,4358	0,4384
26	0,4384	0,4410	0,4436	0,4462	0,4488	0,4514	0,4540
27	0,4540	0,4566	0,4592	0,4617	0,4643	0,4669	0,4695
28	0,4695	0,4720	0,4746	0,4772	0,4797	0,4823	0,4848
29	0,4848	0,4874	0,4899	0,4924	0,4950	0,4975	0,5000
30	0,5000	0,5025	0,5050	0,5075	0,5100	0,5125	0,5150
31	0,5150	0,5175	0,5200	0,5225	0,5250	0,5275	0,5299
32	0,5299	0,5324	0,5348	0,5373	0,5398	0,5422	0,5446
33	0,5446	0,5471	0,5495	0,5519	0,5544	0,5568	0,5592
34	0,5592	0,5616	0,5640	0,5664	0,5688	0,5712	0,5736
35	0,5736	0,5760	0,5783	0,5807	0,5831	0,5854	0,5878
36	0,5878	0,5901	0,5925	0,5948	0,5972	0,5995	0,6018
37	0,6018	0,6041	0,6065	0,6088	0,6111	0,6134	0,6157
38	0,6157	0,6180	0,6202	0,6225	0,6248	0,6271	0,6293
39	0,6293	0,6316	0,6338	0,6361	0,6383	0,6406	0,6428
40	0,6428	0,6450	0,6472	0,6494	0,6517	0,6539	0,6561
41	0,6561	0,6583	0,6604	0,6626	0,6648	0,6670	0,6691
42	0,6691	0,6713	0,6734	0,6756	0,6777	0,6799	0,6820
43	0,6820	0,6841	0,6862	0,6884	0,6905	0,6926	0,6947
44	0,6947	0,6967	0,6988	0,7009	0,7030	0,7050	0,7071

Note: Further specifications may be looked up in a table book.

CHAPTER 16 INCLINATION - TAPER

A change of height in relation to the length of a workpiece is called inclination.

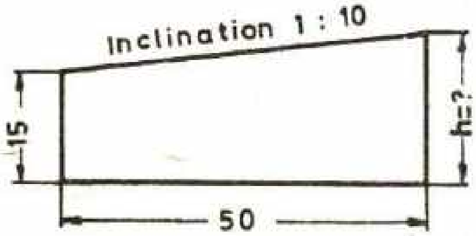
This statement applies for workpieces of which only one side is inclined.



A change of diameters (cone) or lengths of sides (frustum of pyramid) in relation to the length is called taper.

This statement applies for workpieces of which two opposite sides are inclined.

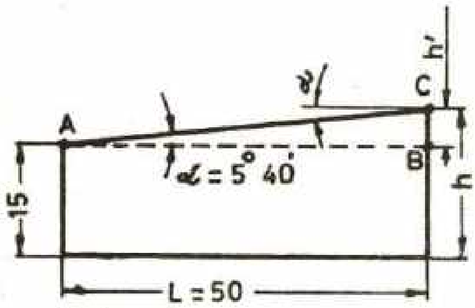
16.1 Inclination



An inclination ratio of 1:10 means that the height of the workpiece changes by 1 mm per 10 mm length. As the total length here is 50 mm, the change will be $\frac{50}{10}$ mm = 5 mm

Total height :

h = 15 mm + 5 mm = <u>20 mm</u>



Note: α = α

If there is no inclination ratio given, the size of "h" can still be calculated, provided the angle of the inclination is given.

For calculating "h" we have to consider ABC as an rectangular triangle, size "h" as the perpendicular and size "L" as the base.

When adding "h'" and the length of the small side (15) we get height "h".

For the given workpiece the way of calculating the total height "h" will be:

Given: base = 50 mm; angle α = 5°40'; small height = 15 mm.

Solution:

a) Finding the size of "h'"

$$\tan \alpha = \frac{\text{perpendicular (h')}}{\text{base (L)}}$$

$$h' = \tan \alpha \times L$$

$$= \tan 5^\circ 40' \times 50$$

$$= 0.0992 \times 50 = 4.96$$

$$\cong \underline{5 \text{ mm}}$$

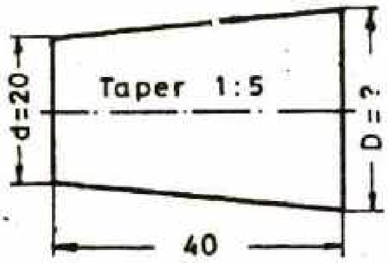
b) Finding the size of "h"

$$h = \text{small height} + \text{"h''"}$$

$$= 15 + 5$$

$$= \underline{\underline{20 \text{ mm}}}$$

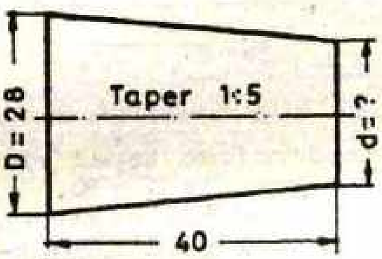
16.2 Taper



A taper ratio of 1 : 5 means that the diameter of a round workpiece changes by 1 mm per 5 mm length.

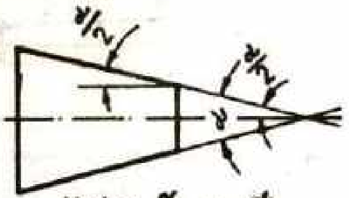
As the total length here is 40 mm, the big dia "D" will be:

$$D = d + \left(\frac{40}{5} \times 1 \text{ mm}\right)$$
$$= 20 \text{ mm} + 8 \text{ mm} = \underline{28 \text{ mm}}$$



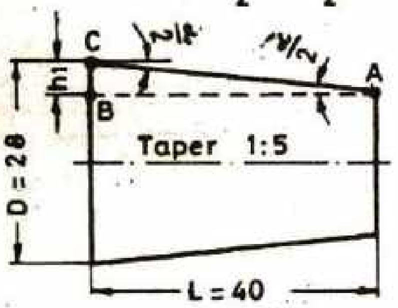
When turning a cone, generally the big dia is given and the small dia has to be found.
In case that the taper ratio is given we calculate:

$$d = D - \left(\frac{40}{5} \times 1 \text{ mm}\right)$$
$$= 28 \text{ mm} - 8 \text{ mm} = \underline{20 \text{ mm}}$$



Note: $\frac{\alpha}{2} = \frac{\alpha}{2}$

For setting the lathe we do need the size of the setting angle. This angle is half the size of the total top angle.



The setting angle $\frac{\alpha}{2}$ must be calculated when not already given in the drawing. This can be done similarly to the method of calculating the inclination.

Again we have to consider size L as the base and size h' as the perpendicular of a rectangular triangle.

Step 1: When not already given we calculate the small dia "d" by using the taper ratio.

$$d = D - \left(\frac{40}{5} \times 1 \text{ mm}\right) = 28 \text{ mm} - 8 \text{ mm} = \underline{20 \text{ mm}}$$

Step 2: Now we have to calculate the size of the perpendicular.

$$2 \times h' = D - d; \; h' = \frac{D - d}{2} = \frac{28 - 20}{2} = \underline{4 \text{ mm}}$$

Step 3: By using trigonometrics we find $\alpha/2$:

$$\tan \frac{\alpha}{2} = \frac{\text{perpendicular (h')}}{\text{base (L)}} = \frac{4 \text{ mm}}{40 \text{ mm}} = 0.1$$

Step 4: The value of tan 0.1 will be looked up in the table.

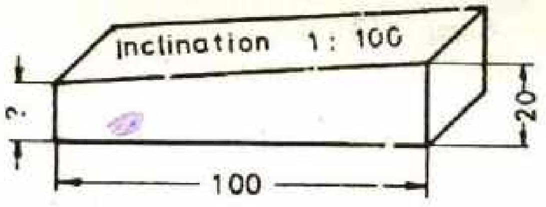
$$\tan 0.1 = \underline{5^\circ 40'} \qquad \alpha = \underline{\underline{5^\circ 40'}}$$

Exercises:

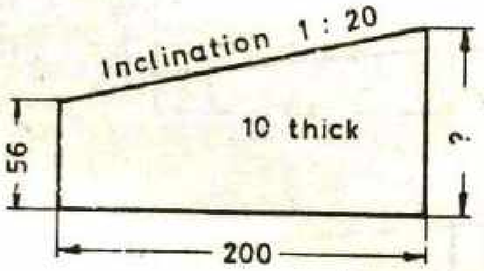
1) Read from the table the values of

a) sin 14° b) tan 25° c) sin 42° d) tan 42°

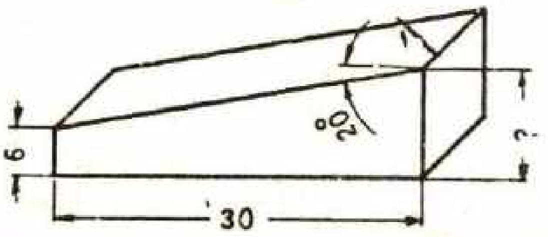
2) The standardized inclination of taper keys is 1 : 100. Calculate the missing height.



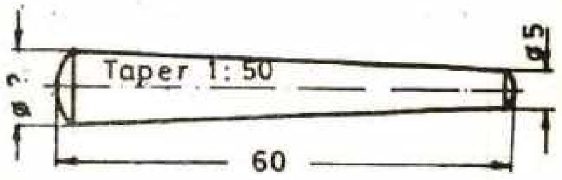
3) The height of the cover sheet is to be calculated.



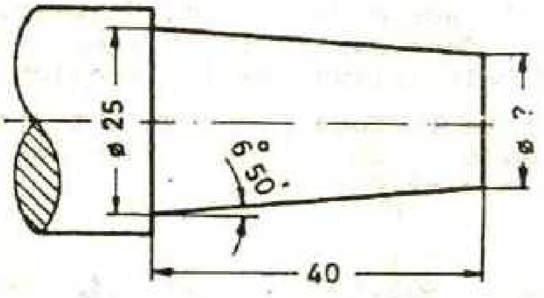
4) Calculate the height of the steel block.



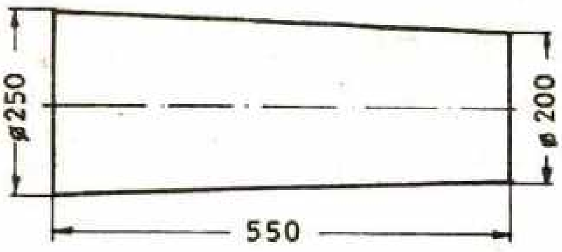
5) The standardized taper ratio of taper pins is 1 : 50. Calculate the big dia of the taper pin 5 x 60.



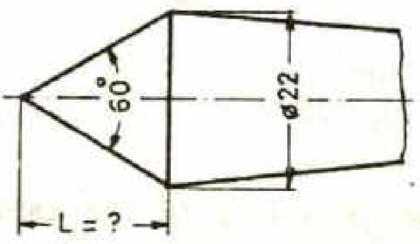
6) Calculate the small dia of the given conic shaft-end.



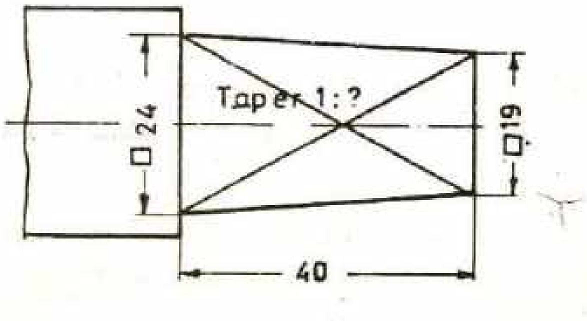
7) Calculate the setting angle.



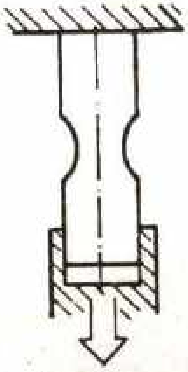
8) Calculate the length L of the lathe centre.



9) Calculate the taper ratio.



CHAPTER 17 STRENGTH



When a force is applied to a workpiece, the internal structure of the material is in a state of stress. The type of material and the dimensions of the workpiece must be selected according to the type and amount of stress expected.

The type of stress depends on how and where the workpiece is used, e.g. whether it is under pressure or whether it is being torn.

The amount of stress depends on the size of the acting force which is applied to the workpiece.

17.1 Force and its units

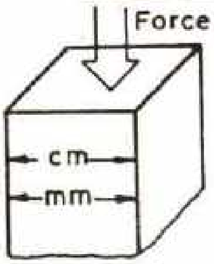
As already pointed out in chapter 12, the force is expressed in Newton (N). Its exact definition is:

1 Newton is the force which gives a mass of 1 kilogramme an acceleration of 1 meter per second squared.

Simplified we can state, that a force of approx. 10 Newton is equivalent a weight of 1 kilogramme.

$$\boxed{1 \text{ kg} \cong 10 \text{ N}}$$

17.2 Force per unit area (pressure)



A force acting on a workpiece is applied to a certain cross-sectional area.
Depending on the units of area used we express pressure this way

$$\boxed{\frac{\text{Force (F)}}{\text{Area (A)}} = \frac{\text{N}}{\text{mm}^2} \quad (\sigma)}$$

If we have to transform one unit into the other, we have to proceed as follows:

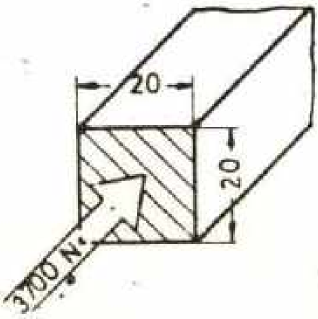
$$\boxed{100 \frac{\text{N}}{\text{cm}^2} = 1 \frac{\text{N}}{\text{mm}^2}}$$

Example: Calculate the force per square centimetre (F/cm^2) and per square millimetre (F/mm^2).

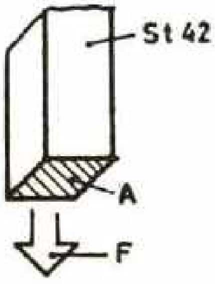
Given: Force (F) = 3700 N
Area (A) = 4 cm^2 = 400 mm^2

Solution a): F/cm^2 = 3700 N : 4 cm^2
= 925 N/cm^2
=========

b): F/mm^2 = 3700 N : 400 mm^2
= 9.25 N/mm^2
==========



17.3 Tensile Strength



The tensile strength is defined as the resistance of a material against tensile load.

It is expressed in N/mm^2 or N/cm^2;

Its symbol is σ (pronounced: sigma)

In our example the steel standard St 42 means, that this steel has a resistance against tensile load of 420 N per square millimetre of its cross-sectional area.

Formula: Tensile strength = $\frac{\text{acting force}}{\text{cross-sectional area}}$

$$\boxed{\sigma\ (\frac{N}{mm^2}) = \frac{F\ (N)}{A\ (mm^2)}}$$

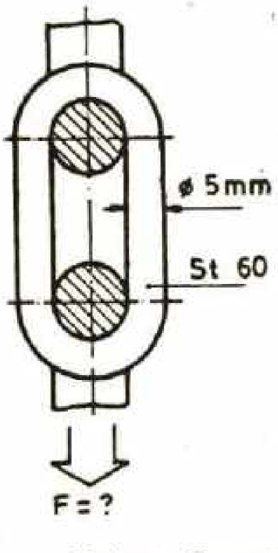
Example:
Calculate the tensile strength "σ" of a material on which a force of F = 24000 N is acting and the cross-sectional area of which is 48 square millimetre.

Given: $F = 24000\ N$; $A = 48\ mm^2$

Solution: $\sigma = \frac{F}{A} = \frac{24000\ N}{48\ mm^2} = \underline{\underline{500\ N/mm^2}}$

A steel St 50 is to be used here.

More frequently than calculating the tensile strength of a material, we have to calculate the maximum tensile load the material of a workpiece or device can bear. In these cases we simply have to transpose the formula.



Example: Calculate the maximum load the chain ø 5 mm, St 60 can bear.
Note that the cross-sectional area has to be taken twice.

Given: $d = 5\ mm$; $\sigma = 600\ N/mm^2$

Solution: a) $A = \frac{d^2 \times \pi}{4} = 19.64\ mm^2$

$A_{total} = 19.64\ mm^2 \times 2 = \underline{39.28\ mm^2}$

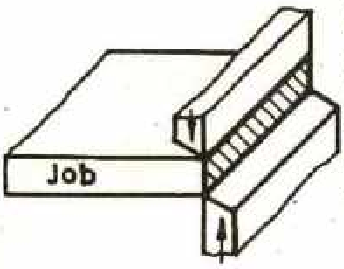
b) $F = \sigma \times A$

$= 600\ N/mm^2 \times 39.28\ mm^2$

$= \underline{\underline{24000\ N}}$

Note: Generally we do not apply the full load as calculated for safety reasons. A factor reducing the calculated load to a safely permitted load has to be taken into account.

17.4 Shearing Strength



The shearing strength is defined as the resistance of a material against cutting and shearing.

It is expressed in N/mm^2 or N/cm^2.

Its symbol is τ (pronounced: tau)

The shearing strength will be calculated from the tensile strength:

$$\tau = 0.8 \times \sigma$$

Example:
Calculate the shearing strength of a material when a force of F = 30 000 N has to be applied for cutting a job with a cross-sectional area of 60 square millimetre.

Note: Step 1: Calculate the tensile strength.
Step 2: Calculate the shearing strength.

Given: $F = 30\,000\ N$; $A = 60\ mm^2$;

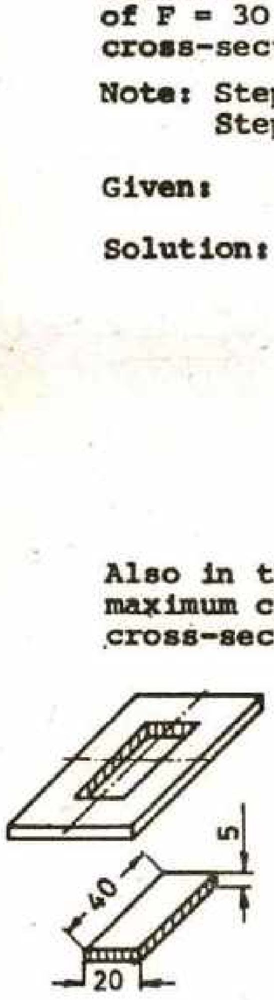
Solution: Tensile strength (σ) $= \dfrac{\text{Acting Force}}{\text{Area}}$

$$= \frac{30\,000\ N}{60\ mm^2} = \underline{500\ N/mm^2}$$

Shearing strength (τ) $= 0.8 \times \sigma$

$$0.8 \times 500\ N/mm^2 = \underline{\underline{400\ N/mm^2}}$$

Also in these cases we are more frequently calculating the maximum cutting force required with the given material and cross-sectional area of a job.



Example: What cutting force F is required to operate the punch of the blanking die?

Note: Step 1: Calculate the cross-sectional area to be cut.
Step 2: Calculate the shearing strength.
Step 3: Calculate the cutting force.

Given: Area A as per drawing.
Tensile strength "σ" $= 340\ N/mm^2$.

Solution:

Step 1: A = cutting perimeter x thickness of the job.
perimeter = 40 + 20 + 40 + 20 = 120 mm

$$A = 120\ mm \times 5\ mm = \underline{600\ mm^2}$$

Step $\tau = 0.8 \times \sigma = 0.8 \times 3\overset{2}{4}0$

$= 272\ N/mm^2$

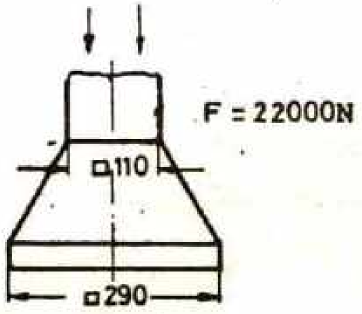
Step $F_{\tau} = \tau \times A = 272 \times 600$

$= 163200N$

<u>Note:</u> Practically we have to apply more cutting force since punch and die never fit so closely that this theoretical value can be obtained.

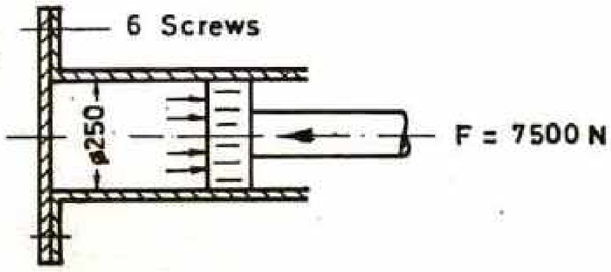
Exercises:

1)



Calculate the pressure per unit area on both the square areas.

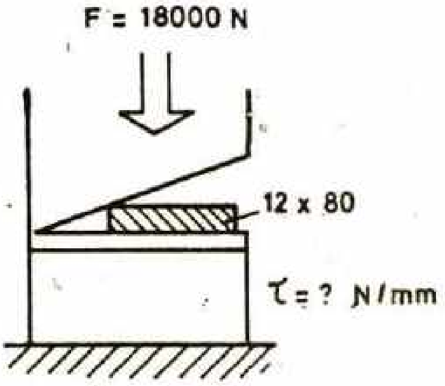
2)



a) Calculate the pressure in N/cm^2 acting at the cover flange.

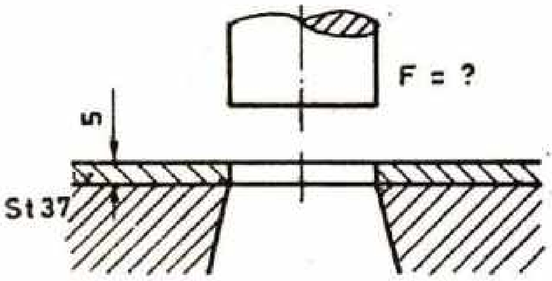
b) Calculate the tensile load stressing each of the six screw bolts.

3)



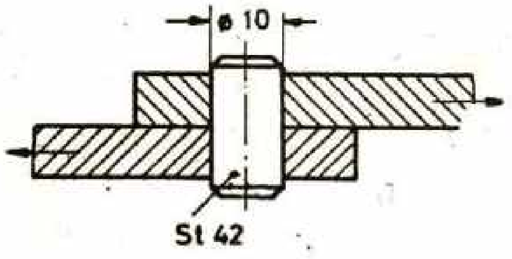
Calculate the shearing strength of the flat.

4)



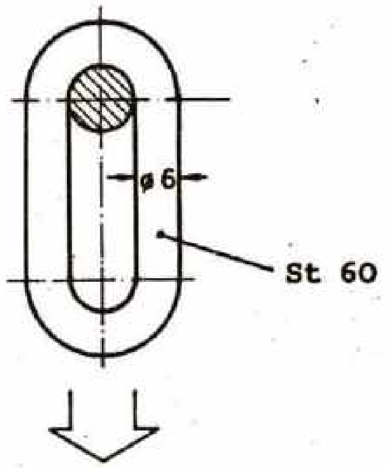
Calculate the punching force required for punching the steel sheet.
Punching dia = 26 mm.

5)



Calculate the maximum force permitted.
Consider a safety factor of 200 %.

6)



Calculate the permitted load considering that only 20 % of the theoretically possible load will be carried.

CHAPTER 18 TIME - MOTION - SPEED

Time is an important factor in technology, e.g.

a pump supplies 450 cubic metre water per hour

a shaft rotates with 205 rotations per minute

the cutting speed of a grinding wheel is 20 metre per second.

18.1 Units of Time

Depending upon the purpose the time factor is used in its different units, which are related to each other as follows:

1 hour (h) = 60 minutes

1 minute (min) = 60 seconds

As can be seen here, contrary to our general system of decimal units, the break-up of time is different. In calculations, however, we sometimes receive results in decimals of a time unit.

Example:
Five workpieces have been drilled in 9 minutes. The time needed for drilling one workpiece, therefore, was 1.8 minutes. To transpose this decimal, 1.8 min, into the time units, i.e. minutes and seconds, we proceed as follows:

1.8 min = 1 minute + 0.8 minute

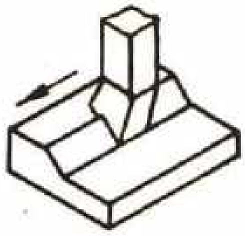
1 minute = 60 sec; 0.1 min = 6 sec

0.8 min = 8 x 6 sec = 48 sec

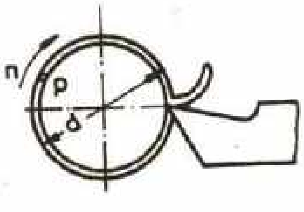
1.8 min = a minute + 48 seconds

18.2 Motion

When a body changes its position it is said to be in motion. There are two types of motions to be dealt with here:



Linear Motion



Angular (Rotary) Motion

Note: The term "motion" simply describes the state of a body. A body is either in a steady state or in a state of motion.

If the distance covered when a body moves and the time in which the distance is covered are involved, we are discussing the speed of a moving body.

18.3 Linear Speed (Velocity)

If a body covers a certain distance in a certain time this body is said to have a speed. The interdependance of speed, distance and time is expressed as follows:

$$\boxed{v = \frac{s}{t}}$$

v = speed (velocity)
s = distance
t = time

Example a) Suppose, a car moves over a distance of 150 km in three hours. What will be its speed?

Given: $s = 150\ km;\quad t = 3\ h;$

Solution: $v = \frac{s}{t} = \frac{150\ km}{3\ h} = \underline{50\ km/h}$

Example b) Suppose, a car is driven with a speed of 65 km/h. What distance will it cover in 4 hours?

Given: $v = 65\ km/h;\quad t = 4\ h;$

Solution: $v = \frac{s}{t};\quad s = v \times t = 65\ km/h \times 4\ h = \underline{260\ km}$

Example c) Suppose, a car is driven over a distance of 120 km with a speed of 50 km/h. What time will be required?

Given: $v = 50\ km/h;\quad s = 120\ km;$

Solution: $v = \frac{s}{t};\quad t = \frac{s}{v} = \frac{120\ km}{50\ km/h} = \underline{2.4\ h}$

$$2.4\ h = 2\ h + 0.4\ h = 2\ h + 4 \times 6\ min = \underline{2\ h\quad 24\ min}$$

Since time and distance are expressed in different units (or parts thereof), the denomination of the speed will be expressed accordingly. Generally we use the following denominations:

$$v = \frac{kilometre}{hour} = \frac{km}{h};\qquad v = \frac{metre}{minute} = \frac{m}{min};\qquad v = \frac{metre}{second} = \frac{m}{sec};$$

It is possible to convert from one unit into another, e.g.:

$$1\ km/h = 1000\ m/h \quad or \quad \frac{1000\ m}{60} \times 1\ h = 1.66\ m/min$$

$$\underline{1\ km/h = 1.66\ m/min}$$

Note: 1 hour consists of 60 minutes. When converting, therefore, we have to use the factor 60.

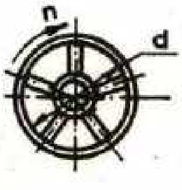
18.4 Rotary Speed

When calculating the rotary speed we basically follow the same procedure used in the case of the linear speed: $v = \frac{s}{t}$

The distance "s", however, has to be found from the development of the circle (= circumference) and the number of revolutions. As the latter will always be given in a certain time, the factor "t" is also included.

Formula: Rotary speed $\boxed{v = d \times \pi \times n}$

Example a) A pulley, dia 0.3 m, is rotating with n = 50 rev/min. Calculate its rotary speed.



Given: $d = 0.3$ m; $n = 50$ rev. per minute;

Solution: $v = d \times \pi \times n$

$= 0.3 \text{ m} \times 3.14 \times 50 \text{ rev/min}$

$= \underline{47.12 \text{ m/min}}$

Example b) Suppose, the dia of the pulley is given in mm, (as is usually done), the result will be:

Given: $d = 300$ mm; $n = 50$ rev/min;

Solution: $v = d \times \pi \times n$

$= 300 \text{ mm} \times 3.14 \times 50 \text{ rev/min}$

$= 47\ 120 \text{ mm/min}$

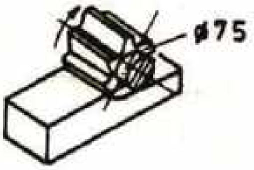
Note: A denomination in mm/min is uncommon. We, therefore, have to convert into m/min. The best way is to do this directly in the formula.

Solution: $v = \frac{d \times \pi \times n}{1000} = \frac{300 \text{ mm} \times 3.14 \times 50 \text{ rev/min}}{1000}$

$= \underline{47.12 \text{ m/min}}$

Note: 1 m = 1000 mm !

Example c) Calculate the number of revolutions of the cutter, when its dia is given as 75 mm and the rotary speed will be 20 m/min.



Given: $d = 75$ mm; $v = 20$ m/min;

Solution: $v = \frac{d \times \pi \times n}{1000}$ $\qquad n = \frac{v \times 1000}{d \times \pi}$

$n = \frac{20 \text{ m/min} \times 1000}{75 \text{ mm} \times 3.14} = \underline{\underline{85 \text{ rev/min}}}$

Example d) Suppose, "n" were given as 120 rev/min. What will be the rotary speed of the cutter now?

Given: $d = 75$ mm; $n = 120$ rev/min;

Solution: $v = \frac{d \times \pi \times n}{1000} = \frac{75 \text{ mm} \times 3.14 \times 120 \text{ rev/min}}{1000}$

$= \underline{\underline{28.26 \text{ m/min}}}$

<u>Remark:</u> The rotary speed of cutting tools is commonly called the cutting speed.

Exercises:

1) Convert into hours and minutes.

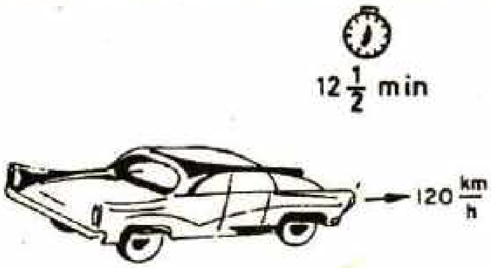
a) 1.5 h b) 2.8 h c) 3.25 h d) 14.2 h

2) Convert into minutes and seconds.

a) 27.3 min b) 16.7 min c) 42.9 min d) 4.75 min

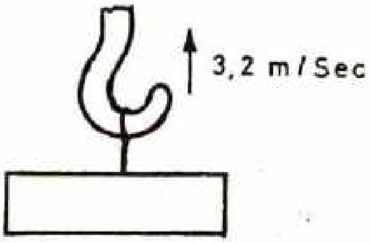
3) For welding 6 workpieces a time of 80 min was needed. Calculate the exact time in min and sec for one job.

4)



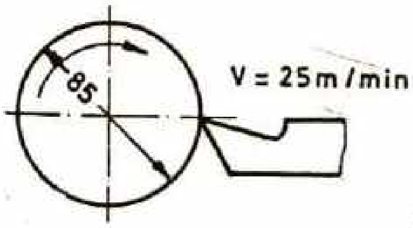
Calculate the distance the car is moving in km and m.

5)



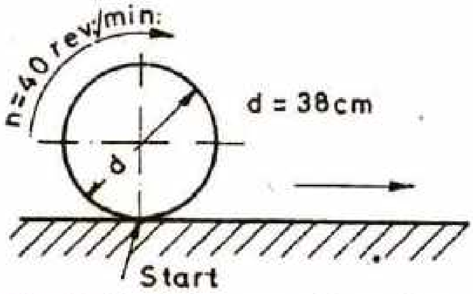
How many metres will the load be lifted in 0.2 min?

6)



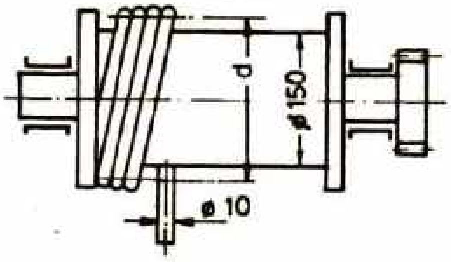
Calculate the number of revolutions per minute.

7)



Calculate the speed and the distance which the wheel moves in 5 min.

8)



a) Determine the number of revolutions per minute required to wind up 26 m of the rope.

b) Calculate the rotary speed of the rope barrel when it does 25 rev/min.

c) Calculate the time required to wind up the rope of 26 m length.

CHAPTER 19 WORK AND POWER

To move a body from one place to another means that we apply a certain force on the body for a certain distance. This is called work.

$$\boxed{\text{work} = \text{force} \times \text{distance}}$$

A work done in a certain time is called power.

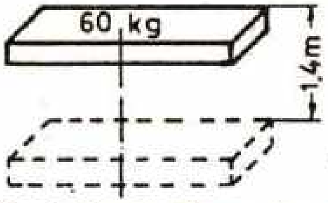
$$\boxed{\text{power} = \frac{\text{work}}{\text{time}} \quad \text{or} \quad \frac{\text{force} \times \text{distance}}{\text{time}}}$$

19.1 Work and its units

Since force is expressed in Newtons and distance in metres (or its units), the denomination of work is expressed accordingly. Generally we express work in Newton x metre (Newton metre). One Newton metre is defined as one Joule and serves as a unit for work:

$$\boxed{1 \text{ Joule} = 1 \text{ Newton} \times 1 \text{ metre} \quad \underline{\text{or}} \quad 1 \text{ J} = 1 \text{ Nm}}$$

Example: What amount of work is required to lift the weight?



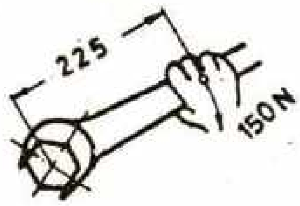
Given: F = 60 kg = 600 N; s = 1.4 m;

Solution: work = force x distance

= 600 N x 1.4 m

= 840 J

19.2 Work done in Rotation



If the force is applied in rotation the distance will be calculated by multiplying the circumference times the number of revolutions:

work = force x distance

distance = circumference x revolutions

$$\boxed{W = F \times d \times \pi \times n}$$

Example: Calculate the work done according to the above sketch. Two revolutions are being made.

Given: Radius = 225 mm; F = 150 N; n = 2 rev;

Solution: $W = F \times d \times \pi \times n$

$d = 2 \times r = 2 \times 225$ mm

$= 450$ mm $= \underline{0.45 \text{ m}}$

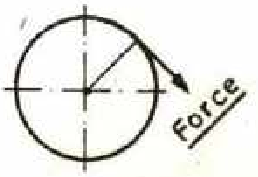
$W = 150 \text{ N} \times 0.45 \text{ m} \times 3.14 \times 2$

$= \underline{\underline{423.9 \text{ J}}}$

Mind: Convert dia 450 mm into dia 0.45 m !

19.3 Torque

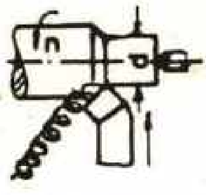
If a work is done in rotation, a force is applied at the end of a lever or at the radius of a round body. This is called Torque.



Torque = force x radius (lever)

Torque is expressed in Joule.
Its symbol is M.

Example a)



The cutting resistance of a material is 5000 N while turning a workpiece, dia 60 mm.
Calculate the torque being transmitted by the motor.

Given: dia = 60 mm → radius = 30 mm

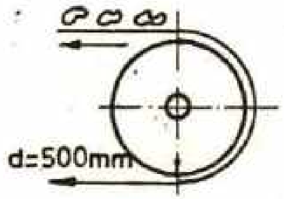
force applied: 5000 N

Solution:

$$M = F \times r$$

$$= 5000\ N \times 0.03\ m = \underline{150\ J}$$

Example b)



The torque of a belt conveyor is given as 1040 J. What is the maximum load the belt conveyor can pull?

Given: M = 1040 J; d = 0.5 m; r = 0.25 m;

Solution: $M = F \times r$

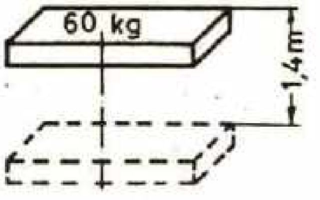
$$F = \frac{M}{r} = \frac{4000\ Nm}{0.25\ m} = \underline{\underline{16\ 000\ N}}$$

19.4 Mechanical Power and its Units

If a mechanical work is done in a certain time it is called mechanical power.
As work is expressed in Joule and time in seconds, minutes or hours, we have to express power accordingly. Generally the power is given as work per second. It is expressed in Watt; its symbol is P.

$\text{power} = \frac{\text{work}}{\text{time}}$. $P = \frac{W\ (J)}{t\ (sec)}$.	$1\ \text{Watt} = \frac{1\ \text{Joule}}{1\ \text{second}}$

Example a)



What power is required to lift the weight in 0.8 seconds?

Given: F = 600 N; s = 1.4 m; t = 0.8 sec;

$$\text{Solution: } P = \frac{W}{t} = \frac{F \times s}{t} = \frac{600\ N \times 1.4\ m}{0.8\ sec}$$

$$= \underline{\underline{1050\ \text{Watts}}}$$

<u>Note:</u> Previously mechanical power was expressed in units of horsepower. Now in accordance with the SI-Standards mechanical power is expressed only in watts.

For conversion:

1 HP = 746 W

Example b) A tube well supplies 40 m^3 of water per hour out of a depth of 8 m. Calculate its power in kW.

Mind: 1 kilowatt (kW) = 1 000 Watts (W)

1 litre water = 1 kg

1 cubic metre water = 1 000 l water

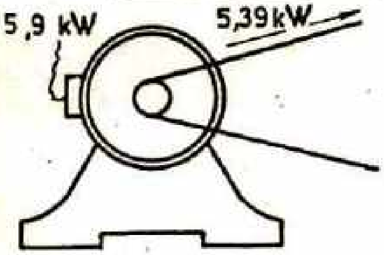
1 hour = 60 min = 3600 seconds

Given: $F = 400\,000$ N; $s = 8$ m; $t = 3600$ sec;

Solution: $P = \frac{F \times s}{t} = \frac{400\,000 \text{ N} \times 8 \text{ m}}{3600 \text{ sec}} = 888.89$ Nm/sec

$= 888.89 \text{ W} = \underline{\underline{0.89 \text{ kW}}}$

19.5 Efficiency



When we operate a machine we put in a certain amount of power and receive back a power in one form or another, e.g. we put electric power in an electric motor and receive a mechanical power.

There are always losses of power during this process, e.g. due to friction. The power output, therefore, is less than the power input.

$$\boxed{\text{Efficiency} = \frac{\text{power output}}{\text{power input}} \quad \text{or} \quad \eta = \frac{P_{out}}{P_{in}}}$$

Example a) Calculate the efficiency of a motor when the power input is 5.9 kW and the power output is 5.39 kW.

Given: $P_{in} = 5.9$ kW; $P_{out} = 5.39$ kW

Solution: Efficiency $(\eta) = \frac{P_{out}}{P_{in}} = \frac{5.39}{5.9} = \underline{\underline{0.913}}$

Note: If multiplied by 100 the efficiency can also be expressed in percentage: 91.3 %

Example b) Calculate the power input of a crane, when a mass of 407 kg is lifted in 1.25 seconds to a height of 2 metres. The efficiency is 80 %.

Given: $F = 4070$ N; $s = 2$ m; $t = 1.25$ sec; $\eta = 0.8$;

Solution: A) $P_{out} = \frac{F \times s}{t} = \frac{4070 \text{ N} \times 2 \text{ m}}{1.25 \text{ sec}}$

$= \underline{\underline{6512 \text{ W}}}$

B) $- = \frac{P_{out}}{P_{in}}$ $\quad P_{in} = \eta \times P_{out} = 0.8 \times 6512$ W

$= 5210 \text{ W} = \underline{\underline{5.21 \text{ kW}}}$

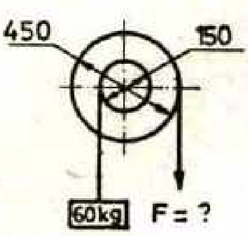
Note: Efficiency is always less than 1 or, in case of percentages less than 100 %.

Exercises:

1) Calculate in Joule the work to be done when a load of 65 kg has to be transported

a) 7.5 m b) 65 m c) 1.76 km d) 220 cm

2)



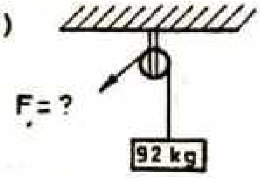
Calculate the force required to lift the load.

Step a) Calculate the anticlockwise torque.

Step b) Calculate the force out of the clockwise torque.

Mind: Clockwise torque and anticlockwise torque must be equal.

3)

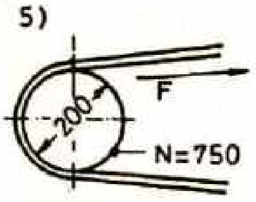


Calculate the mechanical power in kW when the load is to be lifted to a height of 3.5 m in

a) 4 sec b) 0.1 min

4) Calculate the efficiency of an electric motor when the power input is 4 kW and the output is 2 450 J/sec.

5)



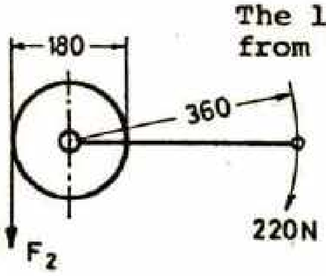
a) What amount of mechanical power in kW is being transmitted by the flat belt?

b) What will be the input of power if the efficiency of the belt conveyor is said to be 0.76 ?

F = 60 000 N; n = 750 rev/min;

6) Calculate the time in which a workshop crane lifts a machine of 1850 kg to a height of 2.3 m.
The motor of the crane supplies a power of 3 kW.
The efficiency of the crane is 78 %.

7)



The load F_2 to be lifted is to be determined from the work W_1 on the crank of a winch.

8) A pump delivers 550 m^3 in 1 hour at a pumping head of 32 m. The efficiency is 0.85.

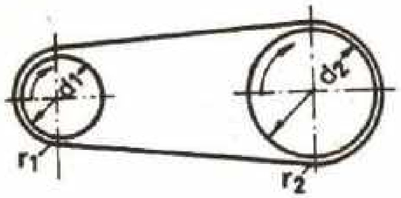
Determine the driving power P_{in} in kW.

CHAPTER 20 FRICTIONAL DRIVE

By means of frictional drive, power can be transmitted from one shaft (the driving one) to another (the driven one).

There are various methods of frictional drives. The belt drive is the most important one.

20.1 Simple Belt Drive



If the two shafts are fitted with pulleys of same diameters the number of revolutions per minute of both shafts will be equal, $n_1 = n_2$.

If the dias differ the rpm's also differ.

The belt speed always remains the same.

Rotary speed of pulley one = Rotary speed of pulley two

$\pi \times d_1 \times n_1$ $\pi \times d_2 \times n_2$

After dividing by the common factor we get:

Formula 1: $\boxed{d_1 \times n_1 = d_2 \times n_2}$ = belt drive formula

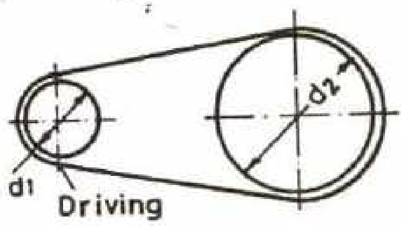
Example a)
The belt pulley of a fan has a dia of 200 mm, its rpm (rev/min) is 150. The motor pulley has a dia of 100 mm. Calculate its rpm.
Mind: Motor pulley = driving pulley !

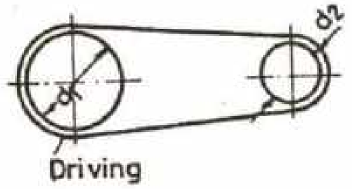
Given: d_1 = 100 mm; d_2 = 200 mm; n_2 = 150 Rpm;

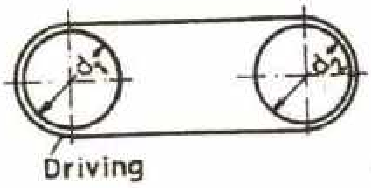
Solution: $d_1 \times n_1 = d_2 \times n_2$

$$n_1 = \frac{d_2 \times n_2}{d_1} = \frac{200 \text{ mm} \times 150 \text{ Rpm}}{100 \text{ mm}} = \underline{300 \text{ Rpm}}$$

When transmitting power through pulleys with different dias, the rpm of the driven shaft is either less (when its pulley is bigger) or more (when its pulley is smaller) than that of the driving shaft.
This change is called the transmission ratio.







The transmission ratio "i" can be calculated either by bringing the dias in relation with each other or by bringing the rpm's in relation with each other.

Formula 2: a) transmission ratio = $\frac{\text{dia driven}}{\text{dia driving}}$ or $i = \frac{d2}{d1}$

b) transmission ratio = $\frac{\text{rpm driving}}{\text{rpm driven}}$ or $i = \frac{n1}{n2}$

Example b)
A motor running at 960 Rpm drives a grinding wheel at 320 Rpm through a belt transmission. Find the transmission ratio.

Given: $n_1 = 960$ Rpm; $n_2 = 320$ Rpm;

Solution: $i = \frac{n_1}{n_2} = \frac{960 \text{ Rpm}}{320 \text{ Rpm}} = \underline{\underline{\frac{3}{1}}}$ <u>or</u> 3 : 1 (three to one)

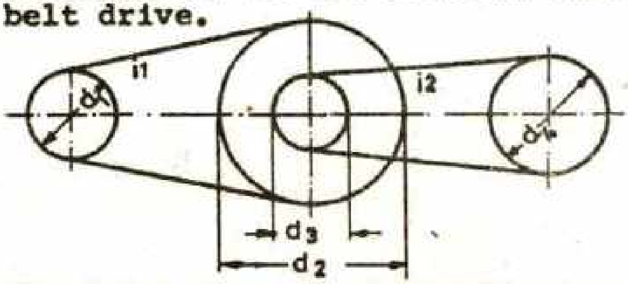
Example c)
In example b) the grinding spindle pulley has a dia of 360 mm. Calculate the size of the motor pulley.

Given: $d_2 = 360$ mm; $i = 3 : 1$;

Solution: $i = \frac{d2}{d1}$; $d_1 = \frac{d2}{i} = \frac{360 \text{ mm}}{\frac{3}{1}} = \underline{120 \text{ mm}}$

20.2 Double belt drive

The size of the permitted transmission ratio of a simple belt drive is limited. If a transmission ratio exceeding this limit is required, we can overcome this problem by using a double belt drive.



A double belt drive can be taken as a combination of two simple belt drives. Pulley nos. two and three are fitted on the same shaft and have, therefore, also the same Rpm.

The total transmission ratio is calculated as follows:

<u>Formula 3:</u>

total transm.ratio = simple trans.ratio x simple transm.ratio

$$\boxed{i_{total} = i_1 \times i_2}$$

Example a)
In a double belt drive we have the following arrangement:

$d_1 = 106$ mm; $d_2 = 530$ mm; $d_3 = 125$ mm; $d_4 = 375$ mm;

motor speed = 1 440 Rpm

Calculate: 1) the two simple transmission ratios
2) the total transmission ratio

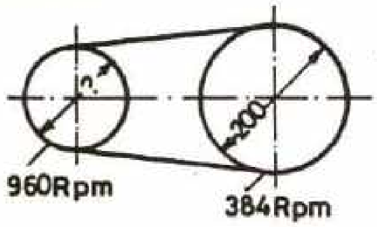
Solution 1) $i_1 = \frac{d2}{d1} = \frac{530 \text{ Rpm}}{106 \text{ Rpm}} = \underline{\frac{5}{1}}$ or $\underline{5 : 1}$

$i_2 = \frac{d4}{d3} = \frac{375 \text{ Rpm}}{125 \text{ Rpm}} = \underline{\frac{3}{1}}$ or $\underline{3 : 1}$

Solution 2) $i_{total} = i_1 \times i_2 = \frac{5}{1} \times \frac{3}{1} = \underline{\underline{\frac{15}{1}}}$ or $\underline{\underline{15 : 1}}$

Exercises:

1)



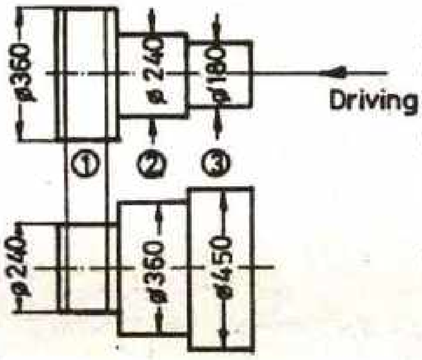
Calculate:

a) The dia of the motor pulley

b) The transmission ratio

2) The transmission ratio of a belt drive is 3:1.
What is the speed of the driven shaft if the driving shaft runs at 1650 Rpm ?

3)



A machine can be operated with three different speeds. The Rpm of the motor is 900.

Calculate:

a) The speeds of the driven shaft in belt position ①, ② and ③.

b) The transmission ratios in the various belt positions.

4) In a Vee-belt transmission the main dia of the driving pulley is 710 mm. What must be the main dia of the driven pulley if a transmission ratio of 2.8:1 is required?

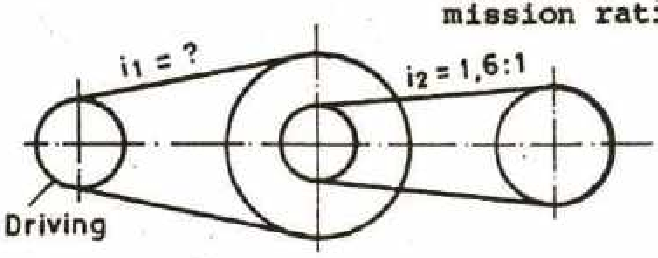
5) In (4) the motor runs at 518 Rpm.
Find the final Rpm by using two different formulas.

6) Calculate the overall transmission ratio

a) $i_1 = 2:1$; $i_2 = 3:1$; b) $i_1 = 3:1$; $i_2 = 2:3$;

7)

Calculate i_1 when the overall transmission ratio is given with 2.4:1.



8) What will be the final shaft speed if the motor in 7) runs with 540 Rpm?

Step 1: Find the Rpm of pulley no. 2 (n_2)

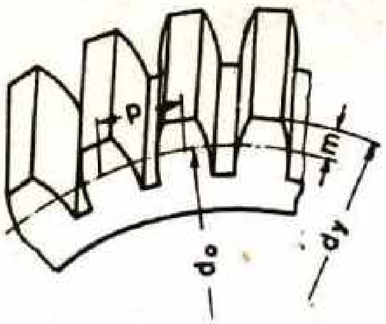
Step 2: Find the Rpm of pulley no. 4 (n_4)

Mind : $n_2 = n_3$!

CHAPTER 21 GEARS AND GEARDRIVE

Gears are used in pairs or in combinations to transmit power from one part of a machine to another part. They are also used to transform motion, change direction of motion and increase or decrease speed.

21.1 Dimensions of Gears



d_o = Pitch dia. It is the dia of an imaginary circle passing through the points at which the teeth of the meshing gears touch each other.

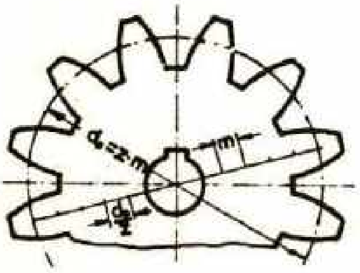
d_y = Outside dia. It is equal to the pitch dia plus two times the module (addendum). It is the major dia at which the gear blank is turned.

m = Module or addendum. It is the portion of the tooth that projects above or outside the pitch circle.

z = Number of teeth of the gear.

p = Circular pitch in mm. It is the distance from the centre of one tooth to the centre of the next consecutive tooth measured on the pitch circle.

21.2 Calculation of Gears



The relations between the various parts of a gear can be derived as follows:

Pitch circle = pitch x number of teeth
Pitch circle = pitch dia x 3.14

$$p \times z = d_o \times \pi$$

The relation $d_o \times \pi = p \times z$ can also be expressed:

$$\frac{d_o}{z} = \frac{p}{\pi} = m \quad \underline{\text{or:}} \quad \frac{d_o}{z} = m; \quad \frac{p}{\pi} = m;$$

This leads to the following formulae:

Formula 1: pitch dia = module x number of teeth $\boxed{d_o = m \times z}$

Formula 2: pitch = module x 3.14 $\boxed{p = m \times \pi}$

As stated before, the outer dia of a spur gear can be calculated.

Formula 3: outer dia = pitch dia + 2 x module $\boxed{d_y = d_o + 2m}$

Example a) Calculate the pitch dia of a spur gear with 26 teeth, module 5 mm.

Given: $z = 26;\quad m = 5\ mm;$

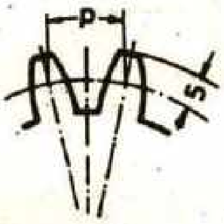
Solution: $d_o = m \times z = 5\ mm \times 26 = \underline{130\ mm}$

Example b) Calculate the number of teeth of a spur gear when the pitch dia is 270 mm with a module = 6 mm.

Given: $d_o = 270\ mm;\quad m = 6\ mm;$

Solution: $d_o = m \times z;\quad z = \frac{do}{m} = \frac{270\ mm}{6\ mm} = \underline{45\ teeth}$

Example c)



Calculate the pitch of the spur gear.

Given: $m = 5\ mm$

Solution: $p = m \times \pi = 5\ mm \times 3.14 = \underline{15.7\ mm}$

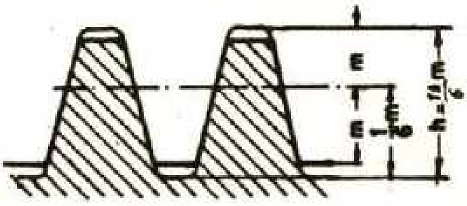
Example d) A spur gear with 36 teeth, module 8 mm will be manufactured. Calculate the outer dimension of the blank.

Given: $z = 36;\quad m = 8\ mm;$

Solution: 1) $d_o = m \times z = 8\ mm \times 36 = 288\ mm$

2) $d_y = d_o + 2m = 288 + 16\ mm = \underline{304\ mm}$

The cutting depth for spur gears and gear racks can be calculated by adding the addendum and the dedendum.



The addendum is the height of the tooth above the pitch circle. It is also called module.
The dedendum is the height of the tooth below the circle line. It is equal to the module plus the clearance.
The clearance will be 1/6 of the module.

$$h = 2m + \frac{1m}{6} = \frac{12+1}{6} x\ m$$

Formula 4:

height of tooth = addendum + dedendum
(=cutting depth)

$$\boxed{h = \frac{13}{6} \times m}$$

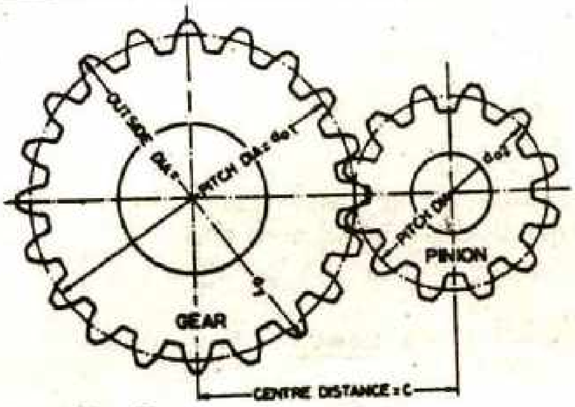
Example e) Calculate the cutting depth for a spur gear with a module of 10 mm.

Given: $m = 10\ mm$

Solution: $h = \frac{13}{6} \times m = \frac{13}{6} \times 10 = \underline{21.67\ mm}$

Mind: The cutting depth must be calculated and set with great accuracy to insure proper running of the gears.

21.3 Centre-to-centre distance



The centre distance is the measurement from the centre of one gear to the centre of the meshing gear.
It is equal to one half of pitch dias of both the gears. This centre distance permits the gears to contact each other at their pitch circles and provides for the smooth and accurate operation of the gears.

$$C = \frac{d_{o_1}}{2} + \frac{d_{o_2}}{2}$$

$$C = \frac{d_{o_1} + d_{o_2}}{2}$$

Formula 5:

$$\text{Centre distance} = \frac{\text{pitch dia 1}}{2} + \frac{\text{pitch dia 2}}{2}$$

Example f) Calculate the centre-to-centre distance of two meshing gears, when their pitch dias are 225 mm and 150 mm respectively.

Given: $d_{o_1} = 225$ mm; $d_{o_2} = 150$ mm;

Solution: $C = \frac{d_{o_1}}{2} + \frac{d_{o_2}}{2} = \frac{225}{2} + \frac{150}{2} = \underline{187.5 \text{ mm}}$

Example g) The measurements of two meshing gears are given as follows:

$z_1 = 50$; $d_{o_2} = 120$ mm; $m = 6$ mm;

Calculate all missing information, that is:
d_{o_1}; z_2; d_y; d_{y_2}; C.

Solutions: $d_{o_1} = m \times z_1 = 6 \text{ mm} \times 50 = \underline{300 \text{ mm}}$

$$z_2 = \frac{d_{o_2}}{m} = \frac{120 \text{ mm}}{6 \text{ mm}} = \underline{20}$$

$$d_{y_1} = d_{o_1} + 2\ m = 300 \text{ mm} + 12 \text{ mm} = \underline{312 \text{ mm}}$$

$$d_{y_2} = d_{o_2} + 2\ m = 120 \text{ mm} + 12 \text{ mm} = \underline{132 \text{ mm}}$$

$$C = \frac{d_{o1}}{2} + \frac{d_{o2}}{2} = \frac{300}{2} + \frac{120}{2} = \underline{210 \text{ mm}}$$

Example h) Suppose, one spur gear out of a gear drive is missing. Calculate its number of teeth when the centre-to-centre distance can be taken as 320 mm, the number of teeth of z_1 is 64 and the module of z_1 is 8 mm.

Given: $z_1 = 64$; $m = 8$ mm; $C = 320$ mm;

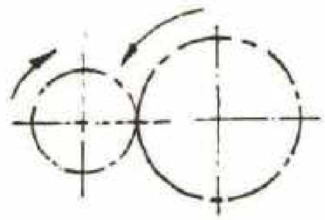
Solution: $C = \frac{d_{o1}}{2} + \frac{d_{o2}}{2}$; $\frac{d_{o1}}{2} = \frac{64 \times 8 \text{ mm}}{2} = \underline{256 \text{ mm}}$

$\frac{d_{o2}}{2} = 320 - 256 = 64$ mm $\qquad d_{o2} = 128$ mm

$$z_2 = \frac{d_{o2}}{m} = \frac{128 \text{ mm}}{8 \text{ mm}} = \underline{16}$$

21.4 Spur Gear Drive (simple gear train)

Two gears in mesh are called a pair of gears. They form the simpliest possible gear train.



The transmission ratio of two meshing gears can be found in the same way as for the belt drive, i.e.:

$$i = \frac{n_1}{n_2} \quad \underline{\text{or}} \quad i = \frac{d_2}{d_1}$$

Mind:

If we express the ratio by using the dias, we have to use the pitch dias since the gears are touching each other there.

Besides these possibilities we can express the transmission ratio by using the gear ratio. The gear ratio of a pair of meshing gears expresses the relationship between the number of teeth each gear contains.

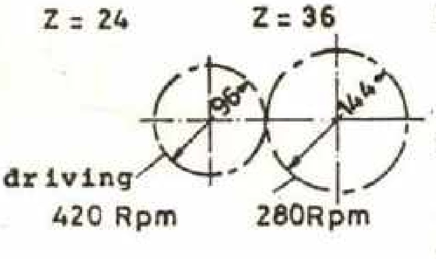
Formula 6:

$$\text{gear ratio} = \frac{\text{number of teeth driven}}{\text{number of teeth driving}} \qquad \boxed{i = \frac{z_2}{z_1}}$$

When calculating simple gear trains we thererore can use

$$\text{either} \quad i = \frac{do_2}{do_1} \quad \underline{\text{or}} \quad i = \frac{n_1}{n_2} \quad \underline{\text{or}} \quad i = \frac{z_2}{z_1}$$

Example i)



$$i = \frac{do_2}{do_1} = \frac{144 \text{ mm}}{96 \text{ mm}} = \frac{1.5}{1} \quad \underline{\text{or}} \quad 1.5 : 1$$

$$i = \frac{n_1}{n_2} = \frac{420 \text{ Rpm}}{280 \text{ Rpm}} = \frac{1.5}{1} \quad \underline{\text{or}} \quad 1.5 : 1$$

$$i = \frac{z_2}{z_1} = \frac{36}{24} = \frac{1.5}{1} \quad \underline{\text{or}} \quad 1.5 : 1$$

Example j) The driven gear has 105 teeth and runs at 145 Rpm. How many teeth has the driving gear when it runs at 725 Rpm ?

Given: $z_2 = 105$; $n_2 = 145$ Rpm; $n_1 = 725$ Rpm;

$$n_1 \times z_1 = n_2 \times 2$$

$$Z_1 = \frac{n_2 \times 2_2}{n_1}$$

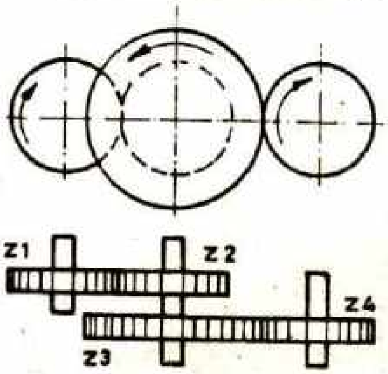
$$= \frac{145 \times 105}{725}$$

The driving gear must have 21 teeth

$$= \underline{\underline{21}}$$

Note: In this calculation the module of the gears is not needed.

21.5 Compound Gear Train



Compound gearing permits a greater range of gear ratio combinations within a limited space than is possible for simple gear trains.
A compound gear train requires at least four gears with a driving and a driven gear at the end of the train and two intermediate gears mounted on the same shaft. It is possible to add one or more sets of intermediate gears thereby enlarging the gear train and number of possible combinations.

As a general rule the overall transmission ratio is calculated by multiplying the gear ratios of each pair of gears.

Formula 7:

gear train ratio = gear ratio x gear ratio

$$\boxed{i_{total} = i_1 \times i_2}$$

Example k) In a gear train the gears have the following toothing: $z_1 = 22$; $z_2 = 44$; $z_3 = 40$; $z_4 = 60$;
Find the overall gear train ratio.

Solution: $i_1 = \frac{z_2}{z_1} = \frac{44}{22} = \underline{\frac{2}{1}}$ $\quad i_2 = \frac{z_4}{z_3} = \frac{60}{40} = \underline{\frac{1.5}{1}}$

$$i_{total} = i_1 \times i_2 = \frac{2}{1} \times \frac{1.5}{1} = \frac{3}{1} \quad \underline{\text{or}} \quad 3:1$$

Since the simple gear ratios can also be obtained from the number of revolutions, i.e. $i = \frac{n_1}{n_2}$,

the overall gear train ratio may also be calculated as a product of these.

Formula 8: $\boxed{i_{total} = \frac{n_1}{n_2} \times \frac{n_3}{n_4}}$

Since $n_2 = n_3$ we can form the relation of

Formula 9:

gear train ratio = $\frac{\text{Rpm of driving gear}}{\text{Rpm of driven gear}}$

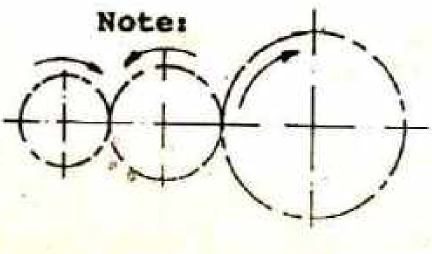
$$\boxed{i_{total} = \frac{n_{start}}{n_{end}}}$$

Example l) The driving gear of a gear train with an overall transmission ratio of 9:1 is running at 1665 Rpm. What will be the Rpm of the last driven gear?

Given: $i_{total} = 9:1$; $n_1 = 1665$ Rpm;

Solution: $i = \frac{n_{start}}{n_{end}}$ $\quad n_{end} = \frac{n_{start}}{i} = \frac{1665}{9} = \underline{\underline{185 \text{ Rpm}}}$

Note:



A gear drive with an idle wheel is not considered a compound train. An idle wheel simply serves the purpose of changing the direction, i.e. it causes the driven gear to rotate in the same direction as the driver.

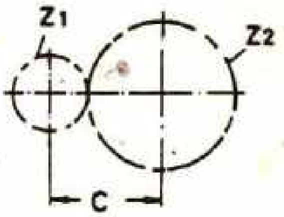
Exercises:

1) A gear with module 12 has 48 teeth.
Find: a) pitch dia b) outside dia.

2) A gear with a pitch dia = 125 mm has 50 teeth.
Find: a) module b) pitch c) cutting depth.

3) A blank has an outside dia of 168 mm. It will have 40 teeth.
Calculate the module.

4)



The driving gear has 36 teeth, the driven one has 54 teeth, the module is 6 mm.

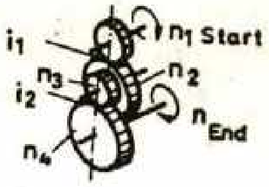
a) Calculate the respective pitch dias.

b) Calculate the centre-to-centre distance.

c) Calculate the gear ratio.

5) The driving gear of No. (4) runs at 1080 Rpm.
Calculate the Rpm of the driven gear.

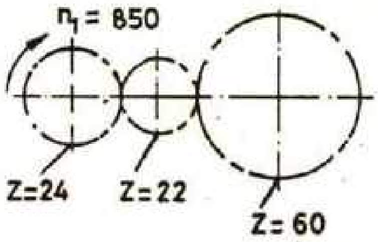
6)



a) Calculate the overall transmission ratio, when $i_1 = 3:2$ and $i_2 = 4:1$.

b) Calculate n_{end} when n_{start} is 1500 Rpm.

7)

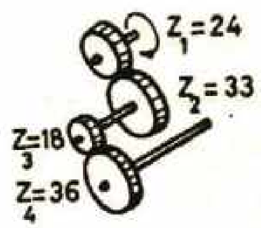


a) Calculate the Rpm of the driven gear.

b) Calculate the pitch diameters when the gear drive has a module = 12 mm.

c) Find the direction in which the driven gear is rotating.

8)



a) Calculate the overall transmission ratio.

b) Calculate the centre-to-centre distance between z_1 and z_2 when the module is 6 mm.

c) What must be the number of rotations of the driver, when z_4 runs at 200 Rpm ?

9) The overall transmission ratio of a gear train is 6.76 : 1.
What is the size of the two equal gear ratios involved?

CHAPTER 22 CUTTING SPEED

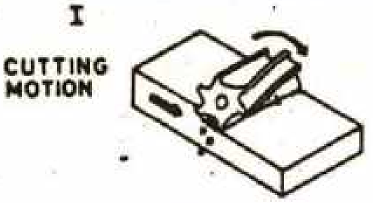
Cutting speed may be defined as the speed with which two parts of a job are being separated from each other, e.g. the speed with which a bolt is turned.
The size of the suitable cutting speed depends on various factors, such as:

- / type of material of the cutting tool;
- / type of material of the cut workpiece;
- / condition of the machine used;
- / type of cutting job, i.e. shaping and milling;
- / construction of the tool;
- / required service life of the tool.

Considering all these factors and based on experience, recommended cutting speeds are given in tables.
For proper setting of the machine, the number of revolutions has to be calculated from these recommended speeds.

22.1 Cutting motions - cutting speed

I



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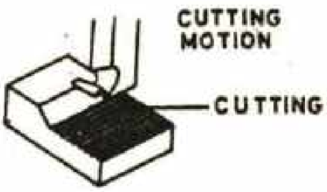


Two main types of cutting motion

circumferential cutting motion

linear cutting motion

As already stated in chapter 18 the speed of a cutting tool is called cutting speed.

The cutting speed of a circumferential cutting job will be calculated:

cutting speed = length of cut x number of revolutions

$$CS\ (m/min) = \frac{d\ (mm) \times \pi \times n}{1000} \qquad CS\ (m/min) = d(m) \times \pi \times n$$

Mind: To achieve a result in m/min we have to divide by 1000 if the dia is given in mm !

Example a) Calculate the cutting speed if a milling cutter, dia 55 mm is rotating with 200 Rpm.

Given: d = 55 mm; n = 200 Rpm;

$$\text{Solution:- } CS = \frac{d \times \pi \times n}{1000} = \frac{55\ mm \times 3.14 \times 200\ Rpm}{1000}$$

$$= 34.5\ m/min$$

Example b) Find out with which Rpm the milling machine has to be set if the disc cutter has a dia of 0.25 m and the cutting speed shall not exceed 30 m/min.

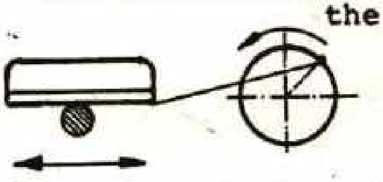
Given: $d = 0.25$ m; $cs = 30$ m/min;

Solution: $CS\ (m/min) = d\ (m) \times \pi \times n$

$$n = \frac{CS\ (m/min)}{d\ (m) \times \pi} = \frac{30\ m/min}{0.25\ m \times 3.14} = \underline{\underline{38\ Rpm}}$$

<u>Note:</u> A Rpm of 38.22, which would be the exact result, cannot be set. We, therefore, have to set 38 Rpm, or the next possible Rpm below 38.22.

For calculating cutting speed which involves a linear motion, the following facts have to be considered:



a) One rotation of the excenter wheel causes a forward stroke and a return stroke. The forward stroke is called the cutting stroke.

b) The lengths of both the strokes have to be added to get the total length.

c) The number of rotations per minute of the excenter wheel includes the time factor.

<u>Formula:</u>

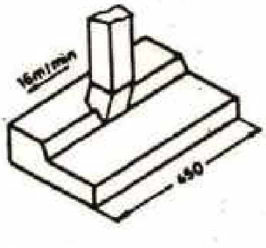
cutting speed = 2 x length of stroke x number of revolutions

$$CS\ (m/min) = \frac{2 \times s\ (mm) \times n}{1000} \qquad CS\ (m/min) = 2 \times s\ (m) \times n$$

Example c) Calculate the Rpm at which the machine has to be set.

Given: $CS = 16$ m/min; $s = 450$ mm;

Solution: $$CS\ (m/min) = \frac{2 \times s\ (mm) \times n}{1000}$$

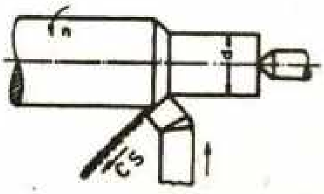


$$n = \frac{CS \times 1000}{2 \times s} = \frac{16\ 000}{900}$$

$$= \underline{\underline{17\ Rpm}}$$

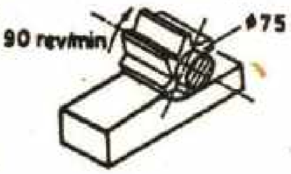
Mind: The machine cannot be set at 17.77 Rpm !

Exercises:



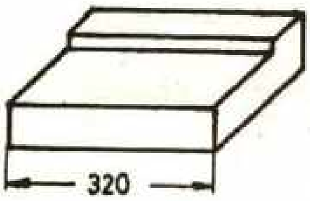
1) A shaft with a dia of 45 mm shall be machined with a cutting speed of 21 m/min. Calculate the suitable Rpm.

2) What will be the maximum dia of a shaft to be machined if the cutting speed is given as 32 m/min and the Rpm as 136 ?



3) Calculate the cutting speed for the milling job.

4) What would be the Rpm of the cutter 3) if a cutting speed of 25 m/min were permitted for the particular material ?



5) The permitted cutting speed for machining the workpiece is 22 m/min. Calculate the number of cutting strokes/min to be set.

   Mind: One cutting stroke per rotation.

6) Suppose, the job is made from different material, i.e. aluminium and copper.

   CS_{copper} = 32 m/min

   $CS_{aluminium}$ = 40 m/min

   Calculate the various Rpm's.

7) Calculate a suitable range of six speeds for a drilling machine, if the size range of the machine is to be for the following drills:

   2.5 mm, 3.25 mm, 4.25 mm, 5.75 mm, 7.5 mm, 10 mm

   and a cutting speed of 22 m/min is to be given.

   Show a speed table with suitable drill size for each speed.

CHAPTER 23 TURNING CALCULATIONS

23.1 Cutting speed, Spindle speed

The cutting speed for turning is the speed of the periphery of the workpiece to be turned.
It is expressed in m/min, its symbol is CS.

The spindle speed of a lathe is the speed at which a workpiece rotates per minute (Rpm), its symbol is n.

Formulae:

$$\text{cutting speed} = \frac{\text{dia of job} \times 3.14 \times \text{spindle speed}}{1000}$$

$$\text{or: } CS\,(\text{m/min}) = \frac{d\ (\text{mm}) \times 3.14 \times n\ (\text{Rpm})}{1000}$$

$$\text{spindle speed} = \frac{\text{cutting speed} \times 1000}{\text{dia of job} \times 3.14}$$

$$\text{or: } n\ (\text{Rpm}) = \frac{CS\ (\text{m/min}) \times 1000}{d\,(\text{mm}) \times 3.14}$$

Table:

According to experience the following cutting speeds are suitable for various materials.
The table also considers the material of the cutting tool and the surface required.
The table is based on an economic service life of 2 hours for the tool.

Table showing Cutting Speeds in m/min of Materials for Turning

Material	HSS Tool		Carbide Tipped Tool		Material	HSS Tool		Carbide Tipped Tool	
	▽	▽▽	▽	▽▽		▽	▽▽	▽	▽▽
Mild Steel	30	45	100	175	Aluminium	60	90	155	305
Tool Steel	25	30	55	90	Copper	55	66	155	215
Stainless Steel	15	25	40	60	Brass	45	60	90	155
Cast Iron (average)	20	25	45	60	Bronze (phosph.)	24	55	90	135
Cast Iron (hard)	12	20	40	55	Plastic	45	75	90	180
					Rubber (hard)	30	55	60	90

Example for reading the table:

Find the suitable cutting speed for turning the following job:
mild steel (ST 34), smoothly machined Tool: HSS

Cutting speed: 45 m/min (from table)

23.2 Feed (for turning)

The feed during turning is the advance travel of the cutting tool during one revolution of the spindle.
It is expressed in mm/rev, its symbol is "s".

Table:

According to experience the following feeds are suitable for various materials and consider the required surface finish and the material of the cutting tool.

Material	HSS Tool		Carbide Tipped Tool	
	▽	▽▽	▽	▽▽
Mild Steel	up to 2.5	up to 0.3	up to 3.5	up to 0.2
Tool Steel	" 1.5	" 0.3	" 1.0	" 0.2
Cast Iron (average)	" 3.0	" 1.5	" 3.0	" 0.2
Cast Iron (hard)	" 2.5	" 1.5	" 2.0	" 0.2
Copper	" 2.5	" 0.3	" 3.0	" 0.2
Brass	" 2.5	" 0.3	" 2.5	" 0.2
Bronze	" 2.5	" 0.3	" 2.0	" 0.2

Note: The suitable feed also depends on the depth of cut and the condition of the machine.

Reading the table

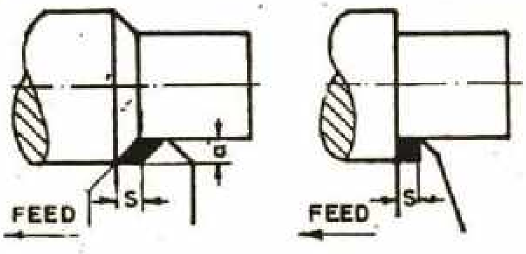
Example: For turning a job, tool steel, smooth surface, a HSS tool is used. Find the suitable feed for setting the lathe.

Solution: The max. feed will be 0.3 mm/rev = 0.3 mm/R

23.3 Depth of Cut, Cross Section of the Chips

The depth of cut is the depth with which the cutting tool penetrates into the workpiece. It is expressed in mm.

The cross section of the chips is the area that is limited by the feed and the depth of cut. It is expressed in square mm.



Note: The cross section of the chips in both cases is the same.

With each cut the dia of the job decreases by twice the depth of cut.

Example: Calculate the depth of cut when a job, dia 55 will be turned to dia 50.
Number of cuts: 1

Solution:

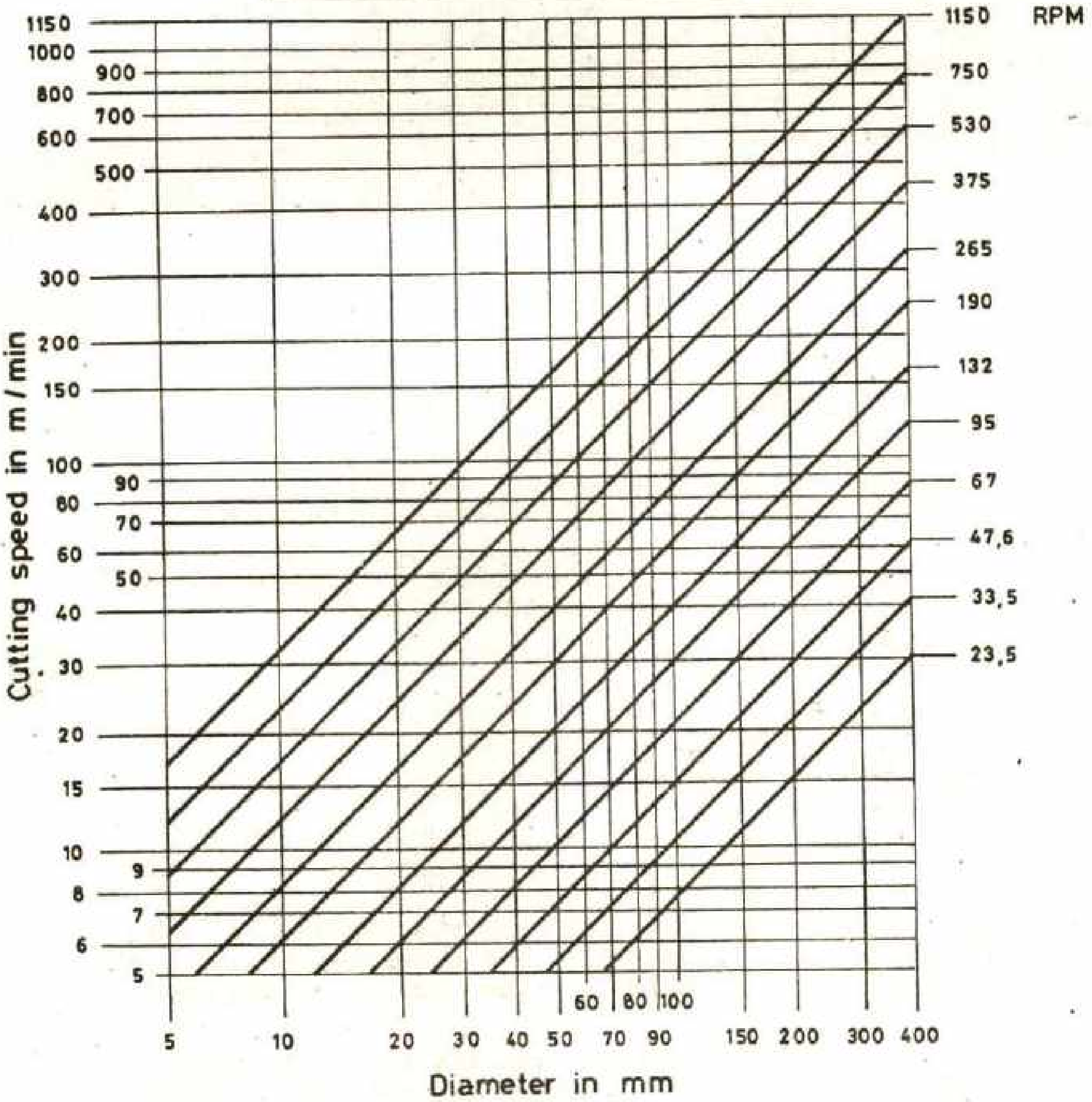
$$\text{depth of cut} = \frac{55\text{ mm} - 50\text{ mm}}{2}$$

$$= \underline{\underline{2.5\text{ mm}}}$$

23.4 Diagram

It is important to set the spindle speed correctly to get good machining results and longer service life out of the tools and machines.
To find out the spindle speed quickly for turning a job of any dia, the cutting speed diagram can be used. From this diagram the spindle speed can be read directly instead of having to calculate it.
One can find these diagrams fixed on the machine.

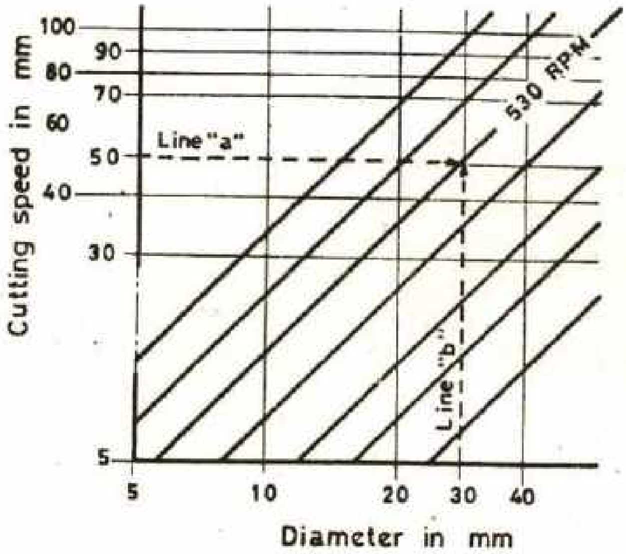
CUTTING SPEED DIAGRAM



The vertical lines show the dia of the workpiece in mm.
The horizontal lines show the cutting speed in m/min.
The inclined lines show the spindle speed to be set.

<u>Note:</u> The given spindle speed can be set at the very machine on which the diagram is fixed.

Reading the Cutting Speed Diagram

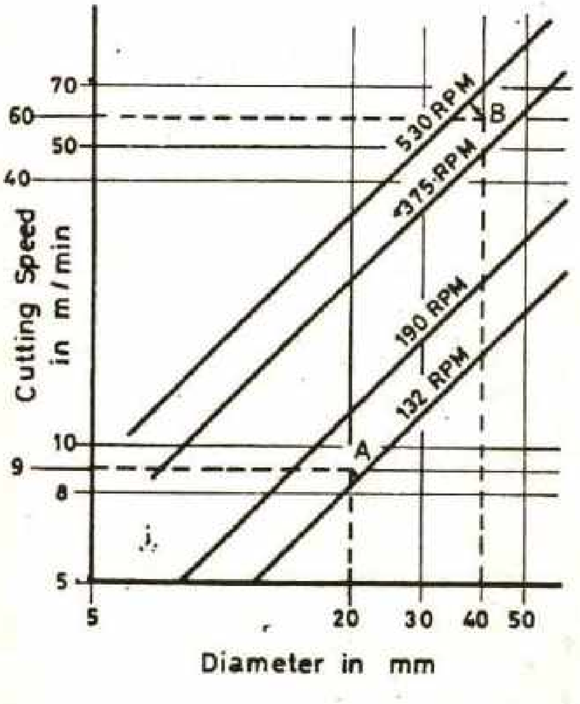


A shaft, dia 30 mm will be turned. The suitable cutting speed has been looked up in the table. It is 50 m/min. The spindle speed required can be read as follows:

a) Mark the position of the cutting speed (50 m/min) on the vertical line and draw a dash-line horizontally (line "a").

b) Mark the position of the dia (30 mm) on the horizontal line and draw a dash-line vertically (line "b").

c) The two lines intersect each other at the 530-Rpm-line.



In these examples the intersection points come in between the spindle speed lines.
In such cases the lower spindle speed lines are selected.

For point 'A' the 132-Rpm-line is selected, for point 'B' it is the 375 -Rpm-line.

If the spindle speed (whether found from the diagram or calculated) does not match with the spindle speed-range of the particular machine, it is preferred to set the machine at the next lower range.

Example: At what speed will a lathe machine be set to turn a cast iron (hard) bar Ø 40 mm, rough surface, if a carbide tipped tool is used?
Speed range of machine: 28-124-220-316-412-508-604 Rpm

Given: d = 40 mm; CS = 40 m/min (from table).

Solution: n = 375 Rpm (from diagram)

This spindle speedcomes in between 316 and 412 Rpm. Therefore, the machine will be set at 316 Rpm.

23.5 Rate of Tool Travel

The rate of tool travel is the movement of the cutting tool along the workpiece per minute.
It is expressed in mm/min. There is no standardized symbol for it. In this book, however, we have given it the symbol TT.

Formula:

Rate of tool travel = feed x spindle speed

or: $TT\ (mm/min) = s\ (mm/R) \times n\ (R/min)$

Example:
For cutting a bolt a lathe is set with a feed of 0.5 mm/R, the spindle speed will be 200 R/min.
Calculate the rate of tool travel.

Given: $s = 0.5\ mm/R$; $n = 200\ R/min$;

Solution: $TT = s \times n = 0.5\ mm/R \times 200\ R/min$

$= \underline{100\ mm/min}$

23.6 Machining Time

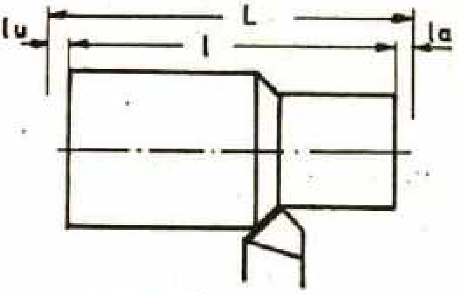
The machining time is the time required for cutting a certain job. It is expressed in minutes, its symbol is t_m.

Formula:

$$\text{Machining time} = \frac{\text{turning length (length of cut)}}{\text{feed/min (rate of tool travel)}}$$

or:

$$t_m\,(min) = \frac{L\ (mm)}{s\,(mm) \times n\,(Rpm)} \qquad t_m = \frac{L\ (mm)}{TT\,(mm/min)}$$



Mind: The turning length "L" includes starting allowance and allowance after turning, l_a and l_u.

Though not fixed, we generally add 5 mm for each allowance to the length of the job.

Example:
A shaft, dia 75 mm, length 270 mm, shall be turned.
Calculate the machining time, when the lathe is set with a feed of 0.2 mm/R and a spindle speed of 285 Rpm.

Given: $d = 75\ mm$; $l = 270\ mm$; $l_a,\ l_u = 5\ mm$; $s = 0.2\ mm$;

Solution: a) $L = l + l_a + l_u = 270 + 5 + 5 = \underline{280\ mm}$

b) $t_m = \frac{L}{s \times n} = \frac{280\ mm}{0.2\ mm \times 285\ Rpm} = \underline{\underline{4.91\ min}}$

23.7 Taper Turning

Taper turning on a lathe machine can be done by employing any of the following methods:

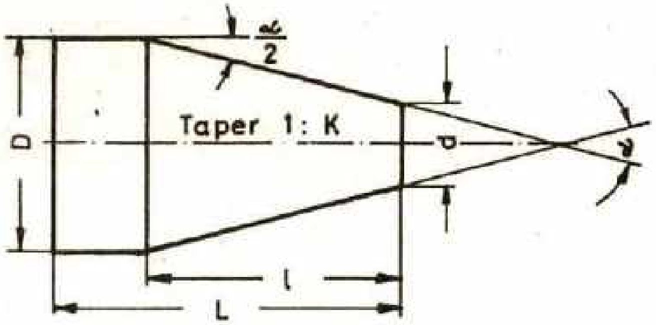
/ tailstock offset method
/ using the compound slide
/ using special taper turning attachment

The necessary calculations vary with the method selected.

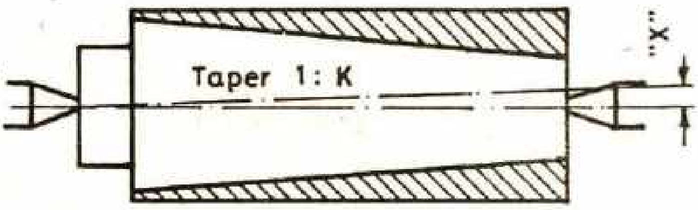
Terminology:

1:K = taper ratio
D = large dia
d = small dia
l = taper length
L = total length
$\alpha/2$ = setting angle
α = taper angle

Mind: 1:K means that the dia changes by 1 mm at a length "K".



A) Tailstock Offset Method



For cutting the taper the centre of the tailstock is brought forward a distance "X", the tailstock offset.

The material removed at the tailstock-end will be twice the amount of the tailstock offset.

Note: This method is useful especially for taper turning of long workpieces, (l $\geq$ 50 x OS).

For calculating the tailstock offset we have to consider the different types of workpieces:

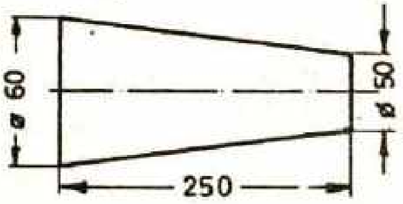
Taper length equal to the total length

Taper length smaller than the total length

Calculation if taper length is equal to total length

Depending on what dimensions are given in the technical drawing we calculate the tailstock offset as follows:

a)

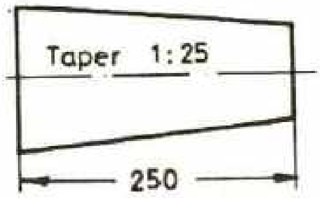


Formula 1: D, d, l are given

$$\boxed{\begin{array}{l} \text{Offset} = \dfrac{\text{big dia - small dia}}{2} \\ \\ \text{or: OS (mm)} = \dfrac{\text{D (mm) - d (mm)}}{2} \end{array}}$$

In this example: $\text{OS} = \dfrac{60 \text{ mm} - 50 \text{ mm}}{2} = \underline{\underline{5 \text{ mm}}}$

b)

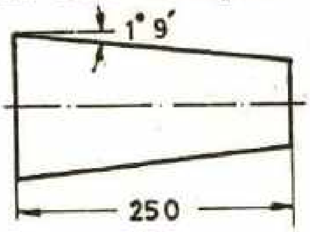


Formula 2: l and taper ratio are given

$$\text{Offset} = \frac{1}{2 \times \text{taper ratio}}$$

or: $$\text{OS (mm)} = \frac{l}{2 \times K}$$

In this example: $$\text{OS} = \frac{l}{2 \times 25} = \frac{250}{50} = 5 \text{ mm}$$



c)

Formula 3: l and setting angle are given

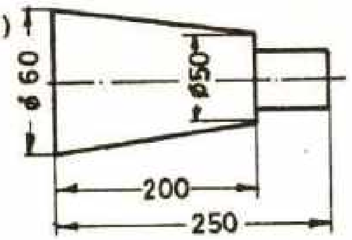
$$\text{Offset} = \text{length} \times \tan \alpha/2$$

or: $$\text{OS (mm)} = l \times \tan \alpha/2$$

In this example: $$\text{OS} = 250 \times \tan 1^\circ 9'$$
$$= 250 \times 0.0202 = 5.05 \text{ mm}$$

Calculation if taper length is smaller than the total length

d)



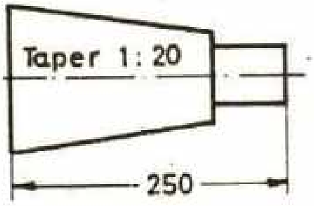
Formula 4: D, d, L, l are given

$$\text{Offset} = \frac{\text{big dia-small dia}}{2} \times \frac{\text{total l.}}{\text{taper l.}}$$

or: $$\text{OS (mm)} = \frac{D - d}{2} \times \frac{L}{l}$$

In this example: $$\text{OS} = \frac{60-50}{2} \times \frac{250}{200} = 6.25 \text{ mm}$$

e)

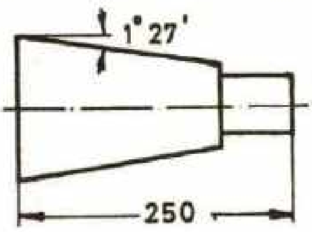


Formula 5: L and taper ratio are given

$$\text{Offset} = \frac{\text{total length}}{2 \times \text{taper ratio}}$$

or: $$\text{OS (mm)} = \frac{L}{2 \times K}$$

In this example: $$\text{OS} = \frac{250}{2 \times 20} = 6.25 \text{ mm}$$



f)

Formula 6: L and setting angle are given

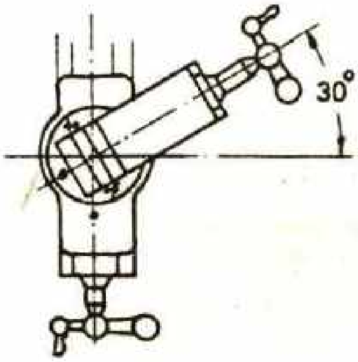
$$\text{Offset} = \text{total length} \times \text{setting angle}$$

or: $$\text{OS (mm)} = L \times \tan \alpha/2$$

In this example: $$\text{OS} = 250 \times 0.0254 = 6.23 \text{ mm}$$

Note: There is a small divergence in the results when we used tangents for calculating the tailstock offset. The reason for this is a small inaccuracy in the value taken from the table. In most of the cases, however, the results are still sufficiently accurate.
Since OS in Ex. d, e and f is bigger than $\frac{L}{50}$ this taper must not be turned by this method.

B) Taper Turning by using the Compound Slide



For cutting the taper the base of the slide is set at the desired setting angle. No automatic feed is possible. The compound slide (rest) must be operated by hand by turning the feed-screw.

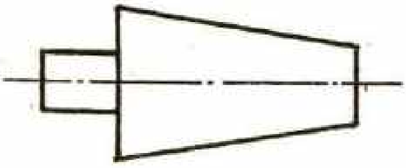
Note: To set the compound slide it is necessary to know either the angle of the taper or the setting angle. Mind that the setting angle is always half the taper angle.

This method is especially useful for steep taper turning of short length.

Formulae: If necessary we can calculate the setting angle, depending on what dimensions are available from the technical drawing, as follows:

Formula 7: taper ratio is given

Formula 8: D, d, l are given

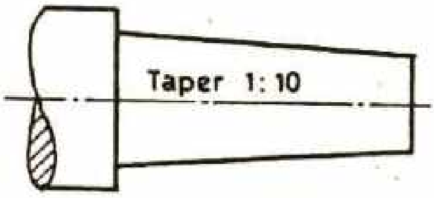


$$\tan\frac{\alpha}{2} = \frac{1}{2 \times \text{taper ratio}}$$

$$\boxed{\tan\frac{\alpha}{2} = \frac{1}{2 \times K}}$$

Example:

Calculate the setting angle!



Given: 1:K = 1:10

Solution:

$$\tan \alpha/2 = \frac{1}{2 \times 10} = 0.05$$

$$\alpha/2 = 2^\circ 52'$$

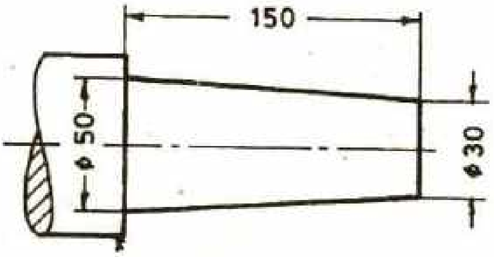
(from table)

$$\tan\frac{\alpha}{2} = \frac{BC}{AB}$$

$$\boxed{\tan\frac{\alpha}{2} = \frac{D - d}{2 \times l}}$$

Example:

Calculate the setting angle!



Given: D = 50 mm l = 150 mm
d = 30 mm

Solution:

$$\tan \alpha/2 = \frac{50 - 30}{2 \times 150} = 0.0666$$

$$\alpha/2 = 3^\circ 48'$$

(from table)

xercises:

) Enter the suitable cutting speeds in the table. (See No. 23.1).

Material of job	Material of tool	Surface finish	Cutting speed
Cast iron (average)	CARBIDE TIP	smooth	m/min
Tool steel	HSS	rough	m/min
Aluminium	HSS	smooth	m/min
Rubber (hard)	CARBIDE TIP	smooth	m/min

2) Find the spindle speeds to be set at the lathe for the above given materials if a bar of Ø 100 mm of each material has to be machined.

   a) by means of calculation

   b) from the diagram (see no. 23.4)

3) Suppose, a copper bar, dia 85 mm, will be turned with HSS tool at a cutting speed of 65 m/min.

   a) What will be the maximum feed permitted to achieve a smooth surface?

   b) What spindle speed has to be set if the speed range of the machine is:

   32-144-242-355-478-592-740 Rpm ?

4) Calculate the missing values.

No:	a	b	c	d	e	f	
d	45	105	60	90	?	?	mm
n	180	60	?	?	190	190	Rpm
CS	?	?	22	22	24	60	m/min

5) A shaft, dia 60 mm, will be turned with a cutting speed CS = 150 m/min. Find the suitable spindle speed.

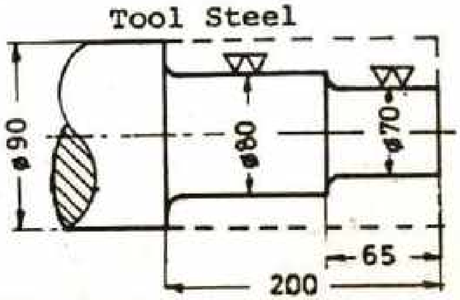
   a) by means of calculation.

   b) from the diagram (see no. 23.4).

   c) Check from the table (no. 23.1) whether the selected cutting speed is within the recommended range if the shaft, St 50, shall be given a smooth surface and a carbide tipped cutting tool will be used.

Exercises:

6)



The end of a shaft will be turned with HSS-tool as given in the sketch.
To obtain the required surface four cuts are required:

cut no. 1: roughing, length 200 mm
cut no. 2: roughing, length 65 mm
cut no. 3: smoothing, length 135 mm
cut no. 4: smoothing, length 65 mm

The feed for roughing has been selected as 1.0 mm/R, that for smoothing as 0.2 mm/R.
The selected cutting speeds are 25 m/min and 30 m/min respectively.

a) Check from the tables whether the selected feeds and cutting speeds are within the recommended ranges.

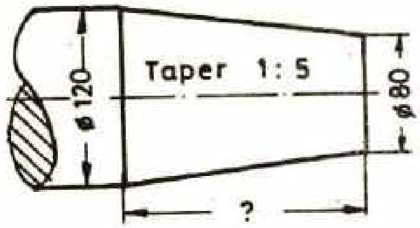
b) Find the suitable spindle speeds for setting the lathe, from the diagram and by calculation.

c) Calculate the machining time for each cut.

d) Calculate the total machining time.

Mind: Only the starting allowance must be considered.

7) A tapered shaft end will be turned.



Calculate:

a) the length "l" of the tapered part

b) the setting angle $\alpha/2$

8) Calculate the setting angles for producing tapers with the following taper ratios:

a) 1:6 b) 1:10 c) 1:15 d) 1:20 e) 1:50

9) Calculate the amount of necessary tailstock offset if the following tapers are to be produced by this method:

No:	a)	b)	c)	d)	e)	f)	
D	80	60	70	48	90	88	mm
d	75	55	64	40	85	85	mm
l	250	200	120	160	150	150	mm
L	300	240	160	200	240	200	mm

Mind: The tailstock offset must not be bigger than 1/50 of the total length of the workpiece.
Check whether all the results fit this rule and, therefore, whether all the tapers can be turned by this method.

CHAPTER 24 DRILLING CALCULATIONS

24.1 Cutting speed, Spindle speed

The cutting speed for drilling is the speed of the periphery of the cutting edge of the drill in use.
It is expressed in m/min, its symbol is CS.

The spindle speed is the speed at which the spindle of the drilling machine rotates per minute (Rpm), its symbol is n.
It can be calculated in the same way as the spindle speed for turning operations.

Formulae:

$$\text{Cutting speed} = \frac{\text{dia of drill} \times 3.14 \times \text{spindle speed}}{1000}$$

$$\text{or: CS (m/min)} = \frac{d\ (\text{mm}) \times 3.14 \times n\ (\text{Rpm})}{1000}$$

$$\text{Spindle speed} = \frac{\text{cutting speed} \times 1000}{\text{dia of drill} \times 3.14}$$

$$\text{or: } n\ (\text{Rpm}) = \frac{\text{CS (m/min)} \times 1000}{d\ (\text{mm}) \times 3.14}$$

Table:

According to experience the following cutting speeds are suitable for various materials to be drilled with HSS-twist drills:

Material to be drilled	Cutting speed (m/min)
Cast iron (average)	15
Cast iron (hard)	9 - 12
Mild steel (up to St 60)	15 - 18
Tool steel	9 - 12
Brass	30 - 40
Bronze	9 - 12
Aluminium	140 - 160

Machine Table for Spindle Speed

According to its construction (transmission ratio !) each drilling machine can be operated generally only with a limited number of spindle speeds. This range is (or should be) marked on a plate fixed on the drilling machine along with instructions on how to set the machine.

Example: A cast iron housing, GG 18, is to be drilled with a drill, dia 20 mm.
Find from table the cutting speed, calculate the spindle speed and point out the actual spindle speed to be set.
Machine table: 47.5-75-118-190-300-475-602-865

Solution: a) CS = 15 m/min (from table)

$$\text{b) } n = \frac{\text{CS} \times 1000}{20 \times 3.14} = \frac{15000}{62.8} = 238.85 \text{ Rpm}$$

c/n (selected) = 190 Rpm (from machine table)

24.2 Feed

Feed is the amount a drill penetrates into the workpiece during one revolution of the spindle.
It is expressed in mm/R, its symbol is s.

Table:

According to experience the following feeds are suitable for various materials to be drilled with HSS-twist drills of different diameters.

Material	Dia of HSS-drill in mm							
	1.5	3	5	10	15	20	25	30
Mild steel	up to 0.1 mm/R			up to 0.4 mm/R				
Tool steel	up to 0.1 mm/R			up to 0.3 mm/R				
Cast iron	up to 0.15mm/R			up to 0.4 mm/R				
Aluminium	up to 0.2 mm/R			up to 0.4 mm/R				
Brass	up to 0.1 mm/R			up to 0.4 mm/R				
Bronze	up to 0.1 mm/R			up to 0.5 mm/R				

Example: Find the suitable feed if a job from tool steel is to be drilled, dia of drill 10 mm.

Solution: The feed will be 0.3 mm/R.

24.3 Rate of Tool Travel

The rate of tool travel is the travel of the drill into the workpiece per minute.
It is expressed in mm/min, its symbol is TT.

Formula:

$$\boxed{TT\ (m/min) = s\ (mm/R) \times n\ (R/min)}$$

Example:

For drilling a steel bar, St 37, the drilling machine has to be set. Drill dia = 16 mm.

a) Find from the table the recommended cutting speed "CS".
b) Find from the table the recommended feed "s".
c) Calculate the spindle speed "n".
d) Find the actual spindle speed from the machine table

35.5-62-128-212-308-426-539-680-796-938-1150

e) Calculate the actual rate of tool travel.

Solution:

a: $CS = \underline{18\ m/min}$ (from table)

b: $s = \underline{0.2\ mm/R}$ (from table)

c: $$n = \frac{18\ m/min \times 1000}{16\ mm \times 3.14} = \underline{358.3\ Rpm}$$

d: $n' = \underline{308\ Rpm}$ (from machine table)

e: $TT = s \times n = 0.2\ mm \times 308\ Rpm = \underline{61.6\ mm/min}$

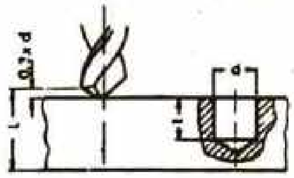
24.4 Machining Time

The machining time is the time required for drilling a certain job. It is expressed in minutes. Its symbol is t_m.

Formula:

$$\text{Machining time} = \frac{\text{drilling length (length of cut)}}{\text{feed / min (rate of tool travel)}}$$

or: $$t_m(\text{min}) = \frac{L\ (\text{mm})}{s\ (\text{mm}) \times n\ (\text{Rpm})} \qquad t_m = \frac{L\ (\text{mm})}{TT\ (\text{mm/min})}$$



Mind: The drilling length "L" includes the starting allowance "l_a".

Generally the starting allowance will be taken as 0.3 x drill dia.

Example a) Calculate the starting allowance if a drill, dia 18 mm, is used.

Solution: $l_a = 0.3 \times 18\ \text{mm} = \underline{5.4\ \text{mm}}$

Example b) A steel plate, 25 mm thick, will be drilled with a twist drill, dia 20 mm. Calculate the drilling length.

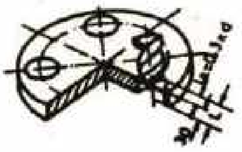
Solution: $L = l + l_a = 25\ \text{mm} + 0.3 \times 20\ \text{mm} = \underline{31\ \text{mm}}$

Example c) A steel cover, St 50, 30 mm thick, will be drilled with a twist drill, dia 22 mm. Calculate the total drilling length. 3 holes will be drilled.

Solution: $L' = 3 \times L$

$L = 30\ \text{mm} + 0.3 \times 22\ \text{mm} = 36.6\ \text{mm}$

$L' = 3 \times 36.6\ \text{mm} = \underline{\underline{109.8\ \text{mm}}}$



Example d) Calculate the machining time required to drill the above steel cover.

Solution:

a) Cutting speed: CS = 17 m/min (from table)

b) Feed : s = 0.2 mm/R (from table)

c) Drilling length: L' = 109.8 mm (from ex. c)

d) Spindle speed : $n = \frac{CS \times 1000}{d \times 3.14} = \frac{17000}{22 \times 3.14}$

$= 246$ Rpm

e) Rate of tool travel: $TT = s \times n = 0.2 \times 246$

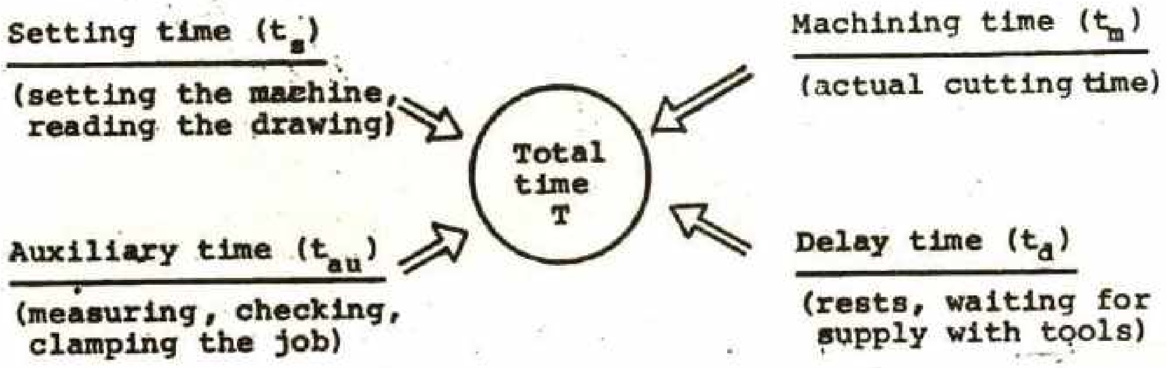
$= 49.2$ mm/min

f) Machining time: $t_m = \frac{L'}{TT} = \frac{109.8\ \text{mm}}{49.2\ \text{mm/min}}$

$= \underline{\underline{2.23\ \text{min}}}$

24.5 Total Time

The total time "T" is the time required for the completion of a work order. Total time consists of different factors which are not only based partly on calculation but also on experience, the condition of tools and machines and the undertaking's organisation.



Setting time (t_s)
(setting the machine, reading the drawing)

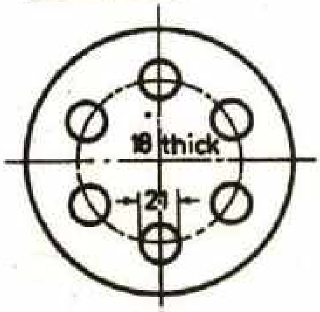
Machining time (t_m)
(actual cutting time)

Total time T

Auxiliary time (t_{au})
(measuring, checking, clamping the job)

Delay time (t_d)
(rests, waiting for supply with tools)

Example:



Calculate the total time if 8 flanges from cast iron are to be drilled.
The setting time is given with 10 minutes, the auxiliary time with 2 min/piece, the delay time will be 15 % of the machining time.

Machine table: 22.5-44-83-185-292-405-625

Mind: To achieve good accuracy the holes will be pre-drilled with dia 5 mm !

Solution:

a) Cutting speed : $CS = \underline{15\ m/min}$ (from table)

b) Feed (dia 5) : $s_1 = \underline{0.15\ mm/R}$ (from table)
(dia 21) : $s_2 = \underline{0.2\ mm/R}$ (from table)

c) Drilling length (1 hole) : $L_1 = 18 + 0.3\ d = 18 + 0.3 \times 5 = \underline{19.5\ mm}$
$L_2 = 18 + 0.3\ d = 18 + 0.3 \times 21 = \underline{24.3\ mm}$

d) Drilling length (1 flange) : $L_1' = L_1 \times 6 = 19.5 \times 6 = \underline{117\ mm}$
$L_2' = L_2 \times 6 = 24.3 \times 6 = \underline{145.8\ mm}$

e) Spindle speed (calculated) : $n_1 = \frac{CS \times 1000}{d_1 \times 3.14} = \frac{15000\ mm}{5\ mm \times 3.14} = \underline{955\ Rpm}$

$n_2 = \frac{CS \times 1000}{d_2 \times 3.14} = \frac{15000\ mm}{21\ mm \times 3.14} = \underline{227\ Rpm}$

f) Spindle speed : $n_1' = \underline{625\ Rpm}$, $n_2' = \underline{185\ Rpm}$ (to be set)

g) Machining time : $t_{m1} = \frac{L_1'}{s_1 \times n_1'} = \frac{117\ mm}{0.15 \times 625} = \underline{1.25\ min}$

$t_{m2} = \frac{L_2'}{s_2 \times n_2} = \frac{145.8\ mm}{0.2 \times 185} = \underline{3.94\ min}$

h) Mach.time/flange: $t_m = t_{m1} + t_{m2} = 1.25 + 3.94 = \underline{5.2\ min}$

i) Delay time/fl. : $t_d = \frac{5.2\ min \times 15\ \%}{100\ \%} = \underline{0.78\ min}$

k) Total time : $T = t_s + 8 \times t_{au} + 8 \times t_m + 8 \times t_d$
$= 10 + 16 + 41.6 + 6.2 = \underline{\underline{73.8\ min}}$

Exercises:

1) Calculate the spindle speed and take from the machine table the speed with which the drilling machine is to be set.

No.:	a	b	c	d	e	f	g	
d (drill)	8	16	24	26	18	12	20	mm
CS	22	22	22	18	26	24	12	m/min
n								Rpm
n								Rpm

Machine table: 26-42-98-175-290-420-630-860-1190

2) Calculate the drilling length!

No.:	a	b	c	d	e	f	g	
d (drill)	12	9	26	12	16	30	8	mm
l (hole)	10	20	20	30	30	10	25	mm
L (total)								mm

3) Calculate the rate of tool travel if

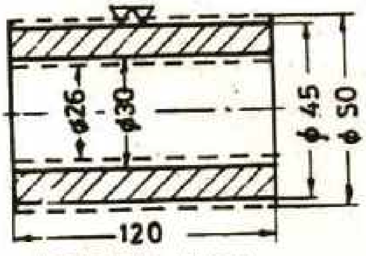
a) s = 0.1 mm/R and n = 276 Rpm

b) s = 0.15 mm/R and CS = 24 m/min. d = 30 mm

4) Calculate the machining time for the following drilling operations:

No.:	a	b	c	d	
L (total)	96	36	120	225	mm
n	660	1200	100	90	Rpm
s	0.16	0.15	0.5	0.4	mm/R
t_m					min

5)



Material: Brass

Calculate:

a) The machining time for turning the bushing.
Mind: Smooth surface means 2 cuts !

b) The machining time for drilling the bushing with a drill ø 30 mm.

c) The total time if only a set-up time of 8 minutes for drilling and turning will be considered.

Turning: l_a, l_u = 5 mm, s (▽) = 0.2 mm/R, s (▽▽)= 0.12 mm/R

CS = 40/60 m/min

CHAPTER 25 MILLING CALCULATIONS

25.1 Spindle Speed

The spindle speed for pulling is the speed at which the spindle of a milling machine rotates per minute. It is expressed in Rpm, its symbol is n.

Formula:

$$\text{Spindle Speed} = \frac{\text{cutting speed x 1000}}{\text{dia of cutter x 3.14}}$$

$$\text{or: n (Rpm)} = \frac{\text{CS (m/min) x 1000}}{\text{d (mm) x 3.14}}$$

Example:

A cutter has a diameter of 80 mm. Find the spindle speed when the cutting speed must not be more than 20 m/min.

Machine table: 16-34-70-155-280-320

Given: d = 80 mm, CS = 20 m/min

$$\text{Solution: } n = \frac{\text{CS x 1000}}{\text{d X 3.14}} = \frac{20\ 000}{\text{80 x 3.14}} = \underline{\text{79.5 Rpm}}$$

n'= 70 Rpm (from machine table)

25.2 Cutting Speed and Feed of Milling Cutters

The cutting speed of a milling cutter is the speed at which the circumference of the cutter passes over the work. It is expressed in metre/min, its symbol is CS.

Formula:

$$\text{cutting speed} = \frac{\text{dia of cutter x 3.14 x spindle speed}}{1000}$$

$$\text{or: CS (m/min)} = \frac{\text{d (mm) x} \quad \text{.14 x n (Rpm)}}{\text{000}}$$

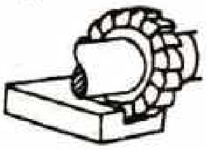
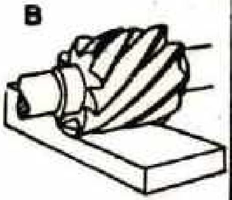
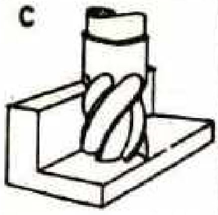
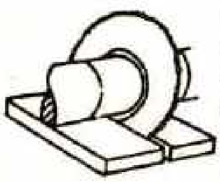
The speed at which the k passes the cutter is called the rate of feed. The amount of feed can be expressed in two ways:
a) in mm/R b) in mm per tooth.
Generally, however, the term "feed" is used, which is the same as the term "rate of feed" or "rate of tool travel", i.e. they are expressed in mm/min.
This is how the feed for milling is expressed.
The symbol for the rate of feed is s'.

Example: Calculate the feed (rate of feed) when the work passes 0.15 mm/R along a cutter with a Rpm of 76.

Solution: feed (rate of feed) = 0.15 mm/R x 76 Rpm
= 11.4 mm/min
==========

Table (cutting speed, feed)

Depending on the shape of the cutter, the type of the cutter and the surface finish, the following cutting speeds and feeds are suitable. Material of cutter: HSS

			Material to be cut			
Type of cutter	Type of work	m/min mm/min	Mild steel	Tool steel	Cast iron	Brass Bronze
A 	∇∇	CS	18 - 22	10 - 14	15 - 18	45 - 60
		s'	25 - 45	10 - 20	30 - 50	50 - 75
	∇	CS	12 - 14	8 - 10	10 - 12	30 - 40
		s'	40 - 75	20 - 30	60 - 90	80 -120
B 	∇∇	CS	18 - 22	10 - 14	14 - 18	40 - 60
		s'	60 - 90	35 - 45	70 - 90	100 -160
	∇	CS	12 - 14	8 - 10	10 - 12	35 - 50
		s'	60 - 80	25 - 35	70 -100	100 -150
C 	∇∇	CS	20 - 22	12 - 14	16 - 18	50 - 60
		s'	60 - 85	30 - 40	70 - 90	100 -150
	∇	CS	12 - 14	8 - 10	10 - 12	30 - 40
		s'	60 - 80	25 - 35	60 - 80	90 -140
D 	∇∇	CS	25 - 30	16 - 20	20 - 25	60 - 80
		s'	50 - 70	22 - 35	50 - 80	100 -160
	∇	CS	15 - 20	10 - 12	12 - 18	45 - 60
		s'	40 - 60	18 - 25	50 - 60	100 -120

A = Plain straight tooth cutter

B = Plain milling helical cutter

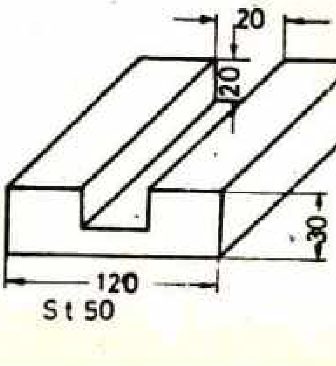
C = Multiple flute end mill

D = Plain metal slitting saw.

Example a) A cast iron workpiece, GG 25, will be cut with a metal slitting saw ø 80 mm. What will be the cutting speed and the feed if a smooth surface is required ?

Solution: CS = <u>22 m/min</u>   s' = <u>60 mm/min</u>   (from table)

Example b) Calculate the spindle speeds to be set, if the given job will be machined.



Solution:

a) Type of cutter: Plain cutter

b) CS (∇) = 13 m/min (from table)

c) CS (∇∇) = 20 m/min (from table)

d) $n_{\nabla} = \frac{CS \times 1000}{d \times 3.14} = \frac{13 \times 1000}{80 \times 3.14} = \underline{\underline{51.7 \text{ Rpm}}}$

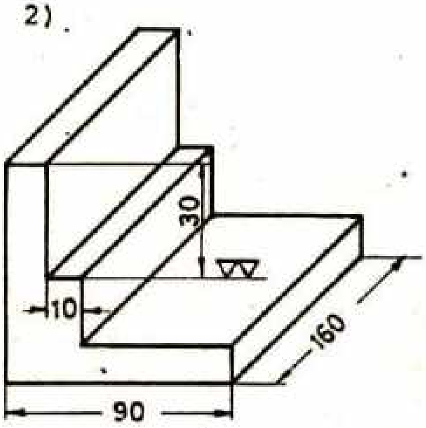
e) $n_{\nabla\nabla} = \frac{CS \times 1000}{d \times 3.14} = \frac{20 \times 1000}{80 \times 3.14} = \underline{\underline{79.4 \text{ Rpm}}}$

Exercises:

1) Enter the missing details for milling the job.

Material of workpiece	Mild steel	Brass	Tool steel
Type of cutter used	Plain straight-tooth cutter	Plain metal slitting saw	Multiple flute end mill
Surface finish of job	smooth	rough	smooth
Cutting speed			m/min
Feed			mm/min

2)



A mild steel bar must be clamped and machined as shown.

a) What type of cutter would be suitable ?

b) Find from the table the suitable cutting speed and calculate the spindle speed if the dia of the cutter is 20 mm.

c) Find the actual spindle speed from the following machine table:

14 - 26 - 42 - 68 - 94 - 155 - 270

d) Calculate the actual cutting speed.

3) A mild steel plate, 80 thick, 60 mm long, will be milled to a thickness of 75 mm, smoothly machined, with a plain helical cutter, dia 100 mm.

Find: a) the number of cuts required considering that the max. cutting depth permitted may be 6 mm for roughing and 1 mm for smoothing.

b) the cutting speeds for both the operations.

c) the rates of feed for both the operations.

d) the spindle speeds for both the operations.

e) the actual spindle speeds to be set according to this machine table:

20 - 45 - 65 - 90 - 160 - 220 - 340.

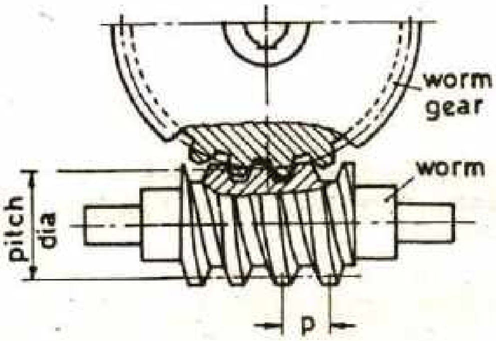
f) the length of cut if a starting allowance and an allowance after milling of together half the cutter dia will be considered.

CHAPTER 26 INDEXING CALCULATIONS

Indexing is used to divide the circumference or the periphery of a job into specified distances or angular separations.

Indexing operations are carried out with a dividing head (indexing head). In its basic construction a dividing head consists of a worm drive.

26.1 Worm Drive



Terms and definitions

Worm: A worm is a cylinder with teeth resembling those of an acme thread.

Pitch: The pitch of a worm is the distance between the centre of one tooth and the centre of an adjacent tooth.

Lead: The lead is the distance a worm (or screw) thread advances axially in one turn.
On a single-thread worm the lead and the pitch are identical; on a double-thread worm the lead is twice the pitch; and so forth.

Pitch and Lead

Worms have a module lead, i.e. their pitch is calculated like that of module gears (see chapter 21).

Formula 1:

pitch = module x π p = m x 3.14

On a single-thread worm the lead is equal to the pitch.
On a multiple-thread worm the lead is a multiple of the pitch.

Formula 2:

lead = pitch x no. of starts = m x 3.14 x Z = p x Z

Example: Calculate the lead of a worm, module 5, if
a) it is a single-thread worm.
b) it is a double-thread (start) worm.

Solution: a) lead = pitch = m x π = 5 mm x 3.14 = 15.7 mm

b) lead = pitch x no. of starts
= 15.7 mm x 2 = 31.4 mm

The advance of a worm is calculated by multiplying the lead with the number of turns (rotations) of the worm.

Formula 3:

Advance = lead x no. of turns = lead x n

Example: Calculate the advance of a worm thread, module 4, triple tnread, if the worm has done 5 revolutions.

Given: m = 4 mm, Z = 3 starts, n = 5 R

Solution: a) lead = p x Z = m x 3.14 x Z = 12.56 x 3 = 37.68 mm

b) advance = lead x n = 37.68 mm x 5 = 188.4 mm

Rpm and Speed Ratio

If a single-thread worm is turned once, the meshing worm gear will advance by one tooth.
If a double-thread worm is turned once, the meshing worm gear will advance by two teeth.

Conclusion: The number of starts of a worm matches with the number of teeth of the worm gear.

For calculating the Rpm and the speed ratio, therefore, the number of starts of the worm can be brought in a relation to the number of teeth of the meshing worm gear.
Since the worm is always the driving part of a worm drive we can state:

Formula 4:

No. of starts of worm x Rpm = No. of teeth of gear x Rpm

or: $Z \times n_1 = z \times n_2$

Example a) Calculate the Rpm of a worm gear, z = 20, if a single-start worm rotates with 120 Rpm.

Given: $Z = 1$, $n_1 = 120$ Rpm, $z = 20$

Solution: $Z \times n_1 = z \times n_2$

$$n_2 = \frac{Z \times n_1}{z} = \frac{1 \times 120 \text{ Rpm}}{20} = \underline{6 \text{ Rpm}}$$

Example b) Calculate the Rpm of a double-start worm if the worm gear, z = 30, rotates with n = 8 Rpm.

Given: $Z = 2$, $z = 30$, $n_2 = 8$ Rpm

Solution: $Z \times n_1 = z \times n_2$

$$n_1 = \frac{z \times n_2}{Z} = \frac{30 \times 8 \text{ Rpm}}{2} = \underline{120 \text{ Rpm}}$$

The speed ratio between worm and worm gear can be calculated as follows:

Formula 5:

$$\text{Speed ratio} = \frac{\text{Rpm of the worm}}{\text{Rpm of the worm gear}} \quad \text{or:} \quad i = \frac{n_1}{n_2}$$

Example a) Calculate the speed ratio of a worm drive if the worm rotates with 40 Rpm and the worm gear with 2 Rpm.

Solution: $i = \frac{n_1}{n_2} = \frac{40 \text{ Rpm}}{2 \text{ Rpm}} = \frac{20}{1}$ or: 20 : 1

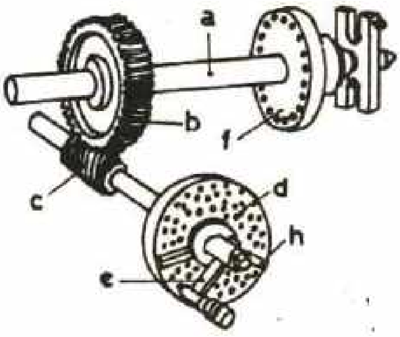
Example b) Calculate the speed ratio if a double-start worm does 60 Rpm and is driving a worm gear with z = 20

Given: $Z = 2$, $n_1 = 60$ Rpm, $z = 20$

Solution: A) $Z \times n_1 = z \times n_2$; $n_2 = \frac{Z \times n_1}{z} = \frac{2 \times 60 \text{ Rpm}}{30} = \underline{4 \text{ Rpm}}$

B) $i = \frac{n_1}{n_2} = \frac{60}{4} = \frac{15}{1}$ or: $\underline{15 : 1}$

26.2 Worm Gearing for Indexing



a = index spindle
b = worm gear
c = worm
d = index plate
e = index crank (handle)
f = index plate for direct indexing

The speed ratio for indexing is generally given as i = 40 : 1 . The number of rotations (turns) of the index crank can be calculated as follows:

Formula 6:

$$\text{No. of turns of index crank} = \frac{\text{speed ratio}}{\text{No. of divisions required}}$$

or:

$$n_c = \frac{40}{N}$$

Example: Index a job for five divisions.

Solution: $n_c = \frac{40}{N} = \frac{40}{5} = 8$ turns of the index crank

26.3 Rapid Indexing

For rapid indexing (also known as direct indexing) the worm drive is disengaged; only the front index plate is used. This index plate usually has 24 equally spaced holes which can be engaged by the front index pin.

The number of holes to move in the index plate can be found by dividing 24 by the number of divisions required.

Formula 7:

$$\text{No. of holes to move} = \frac{24}{\text{No. of divisions required}}$$

or:

$$n_o = \frac{24}{N}$$

Example: Index a hexagon job by rapid indexing.

Solution: No of holes to move $= \frac{24}{N} = \frac{24}{6} = \underline{4}$

Note: There is only a limited number of indexing jobs possible by using this method:

$$\frac{24}{24}; \quad \frac{24}{12}; \quad \frac{24}{8}; \quad \frac{24}{6}; \quad \frac{24}{4}; \quad \frac{24}{3}; \quad \frac{24}{2};$$

26.4 Simple Indexing

Simple indexing (or indirect or plain indexing) is used when a circle must be divided into other parts than is possible by rapid indexing. It requires the use of the worm drive.

A. Number of divisions required divides evenly into 40, i.e. the speed ratio (gear ratio).

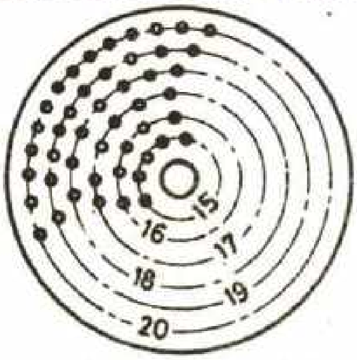
Formula 8 (like formula 5):

$$\text{No. of turns of index crank} = \frac{\text{speed ratio}}{\text{No. of divisions required}}$$

or:

$$n_c = \frac{40}{N}$$

B. Number of divisions required does not divide evenly into 40.
In this case the index crank must be moved fractional parts of a turn. This is done with index plates.



The dividing head generally is furnished with three index plates. Each plate has six circles of holes, as listed:

plate one: 15-16-17-18-19-20

plate two: 21-23-27-29-31-33

plate three: 37-39-41-43-47-49

Example a) Index a job with eighteen divisions.
Mind: This cannot be done by rapid indexing. Nor is the no. of divisions divisible evenly into 40.

Solution: $n_c = \frac{40}{N} = \frac{40}{18} = 2\,\frac{4}{18}$ turns

Note: The whole number indicates the complete turns of the index cranks, the denominator represents the index circle, the numerator gives the no. of holes to use on that circle.

Result: n_c = 2 full turns plus 4 holes on the 18-hole circle.

Example b) Index a job, seven divisions.

Solution: $n_c = \frac{40}{N} = \frac{40}{7} = 5\,\frac{5}{7}$ turns $= 5\,\frac{15}{21}$ turns

Note: When the denominator of the indexing fraction does not match with any of the available index circles, change it to a number represented in one of the circle holes.
Do this by multiplying or dividing the numerator and the denominator by the same number:

$$\frac{5}{7}\,\frac{\times 3}{\times 3} = \frac{15}{21}$$

Exercises:

1) Calculate the missing details of the worm!

No.	a	b	c	d	e	f	g	
No. of starts	1	2		3	1	1		
No. of turns	3	4	1	1	74	5	2	
Module	5	6	8		5		5	mm
Lead			25.12				31.4	mm
Advance of worm				18.84		31.4		mm

2) Calculate the speed ratio and Rpm of the worm drives!

No.	a	b	c	d	e
No. of starts of worm	2	1	3	2	1
No. of teeth of gear	40	25	24	32	60
Rpm of worm	70		96		200
Rpm of gear		10		9	
Speed ratio of drive					

3) Find the number of holes which the index plate has to be moved for rapid indexing if the number of divisions required are as under

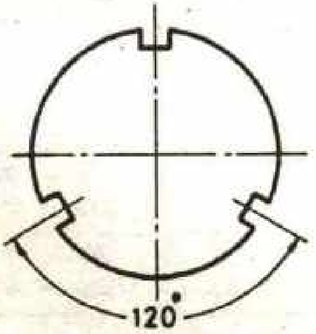
No. of divisions	12	8	2	24	3	6	4
No. of holes							

4) By means of simple indexing a job will be provided with 7 or 9 or 25 divisions.

a) Point out which indexing plates can be used in each case.

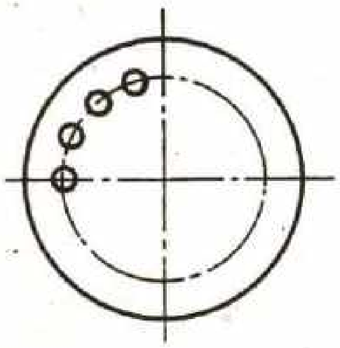
b) Calculate the no. of turns of the crank for each case.

5)



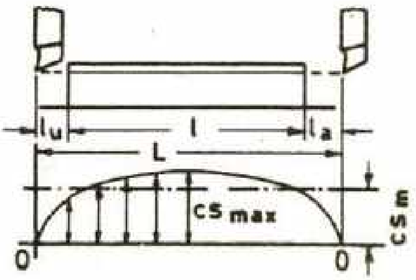
The cover plate will be slotted as drawn. Find the number of full turns of the index crank required and the no. of holes on the 33 circle to be set.

6)



A flange with 35 holes is to be drilled. Find the no. of full turns of the index crank required and the no. of holes to be set.

CHAPTER 27 SHAPING / PLANING CALCULATIONS



The cutting action of the shaper is intermittent:
The tool moves slowly towards the job, increases its speed, slows down again, and returns quickly.

l_a = allowance before start
l_u = allowance after cut

27.1 Cutting Speed for Shaping

The cutting speed for shaping is the speed of the cutting tool in use. For calculation, both the strokes, i.e. cutting and idle strokes, must be considered.
The cutting speed will be taken here as a middled speed.

Formula 1:

$$\text{Cutting speed} = \frac{2 \times \text{length of stroke} \times \text{Rpm of crank gear}}{1000}$$

$$\text{or: } CS\ (\text{m/min}) = \frac{2 \times L\ (\text{mm}) \times n\ (\text{Rpm})}{1000}$$

Example: Find the cutting speed (middled) for a shaping job, if the length of stroke is 320 mm and the crank gear rotates with n = 25 Rpm

Given: L = 320 mm, n = 25 Rpm

Solution: $CS = \frac{2 \times L \times n}{1000} = \frac{2 \times 320\ \text{mm} \times 25\ \text{Rpm}}{1000} = \underline{16\ \text{m/min}}$

Table:

According to experience the following cutting speeds are suitable for various materials to be shaped with HSS or tool steel tools.

	Tool	Mild steel	Cast iron	Tool steel	Brass
∇	Tool st.	10 - 15	8 - 12	8 - 12	15 - 20
	HSS	15 - 20	12 - 16	12 - 16	20 - 25
∇∇	Tool st.	15 - 20	14 - 18	12 - 16	20 - 25
	HSS	20 - 25	18 - 22	16 - 20	30 - 40

27.2 Length of Stroke

The length of stroke of the shaper ram will be set as follows:

Formula 2:

$$\text{Length of stroke} = \text{length of job} + \text{allowance for start \& end}$$

$$\text{or: } L\ (\text{mm}) = l\ (\text{mm}) + l_a\ (\text{mm}) + l_u\ (\text{mm})$$

Note: Generally "l_a" may be considered with 30 mm and "l_u" with 10 mm.

27.3 Number of Strokes per Minute

The number of strokes per minut is equal to the Rpm of the crank gear. It may be calculated as follows:

Formula 3:

$$\text{No. of strokes per min } \underline{\text{or}} \text{ No. of turns of crank gear per min} = \frac{\text{cutting speed} \times 1000}{2 \times \text{length of stroke}}$$

or:

$$n\ (\text{Rpm}) = \frac{\text{CS (m/min)} \times 1000}{2 \times L\ (\text{mm})}$$

Example: A mild steel job, $l = 120$ mm will be machined with a tool steel. Smooth surface required. What will be the setting of the shaping machine?

Machine table: 10-18-28-42-60-86-110

Given: $l = 120$ mm, $l_a = 30$ mm, $l_u = 10$ mm

Solution: a) CS = 15 m/min (from table)

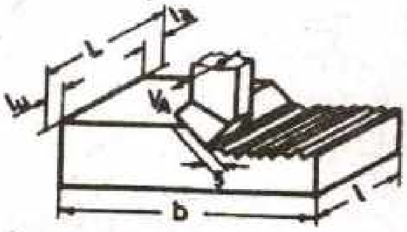
b) $L = l + l_a + l_u = 120 + 30 + 10 = \underline{160 \text{ mm}}$

c) $n = \frac{CS \times 1000}{2 \times L} = \frac{15 \text{ m/min} \times 1000}{2 \times 160 \text{ mm}} = \underline{46.9 \text{ Rpm}}$

$n' = \underline{42 \text{ Rpm}}$ (from machine table)

27.4 Machining Time

The machining time for shaping and planing depends on the feed, the number of strokes per minute, the width of the workpiece and the number of cuts required.



In the drawing shown:

l = length of job

L = length of stroke

s = feed per stroke (cutting stroke)

b = width of job

l_a, l_u = allowances

The machining time for one cut can be calculted as follows:

Formula 4:

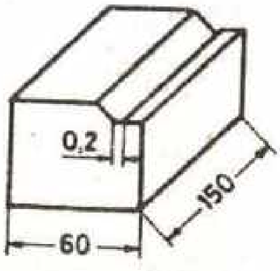
$$\text{Machining time per cut} = \frac{\text{width of job}}{\text{feed} \times \text{no. of strokes p. minute}}$$

or:

$$t_m\ (\text{min}) = \frac{b\ (\text{mm})}{s\ (\text{mm/R}) \times n\ (\text{Rpm})}$$

Is more than one cut required, the result has to be multiplied by the number of cuts.

Example: Calculate the machining time, if a job will be machined with n = 32 Rpm, two cuts.



Given: $l = 150$ mm, $b = 60$ mm, $s = 0.2$ mm, $n = 32$ Rpm

Solution: a) $L = l + l_a + l_u = 150 + 30 + 10 = \underline{190 \text{ mm}}$

b) $t_m = \frac{b}{s x n} = \frac{60 \text{ mm}}{0.2\text{mm} \times 32\text{Rpm}} = \underline{9.4 \text{ min}}$

$t \text{ total} = 2 \times t_m = 2 \times 9.4 = \underline{18.8\text{min}}$

Exercises

1) A planing machine is set as follows:

a) s = 1.2 mm/cutting stroke, n = 32 Rpm
b) s = 0.6 mm/cutting stroke, n = 82 Rpm
c) s = 0.8 mm/cutting stroke, n = 52 Rpm

Find the rate of tool travel in mm/min in each case.

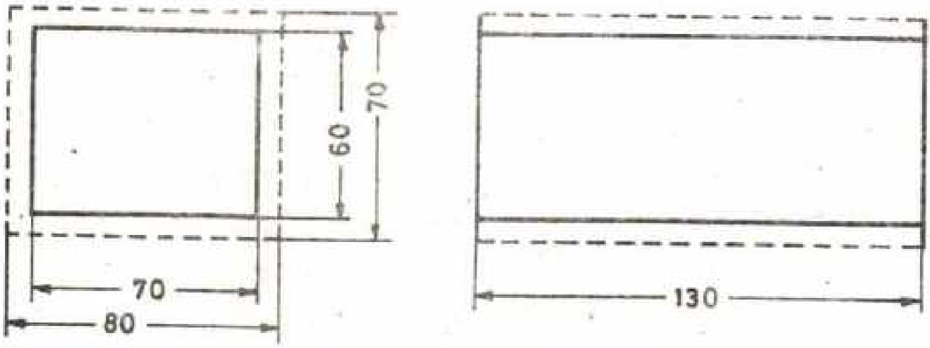
2) Fill in the missing details. Job: smooth surface, tool: HSS

No.	a	b	c	d	
Material of job	Cast iron	Mild steel	Tool steel	Brass	
Length of job	320 mm	0.45 m	220 mm	64 cm	
Length of stroke					mm
Cutting speed (from table)					m/min
Rpm calculated					Rpm
Rpm to be set (from table)					Rpm

Machine table: 6-14-32-52-76-98-122-158

3) Calculate the machining time for the jobs from No. 2) if the width for each job is 100 mm and a feed of 0.2 mm/stroke is given.

4)



A mild steel job, St 42, will be machined from the raw material as shown with a HSS Tool.

a) What will be the setting of the shaping machine? Use machine table from No. 2) with regards to:

Length of stroke? Cutting speed? Rpm to be set?

b) What will be the machining time if one cut for roughing and smoothing each is required?
Feed for roughing: 0.3 mm/R ; Feed for smoothing: 0.2 mm/R
<u>Mind:</u> Add up all single widths thus making a total width.

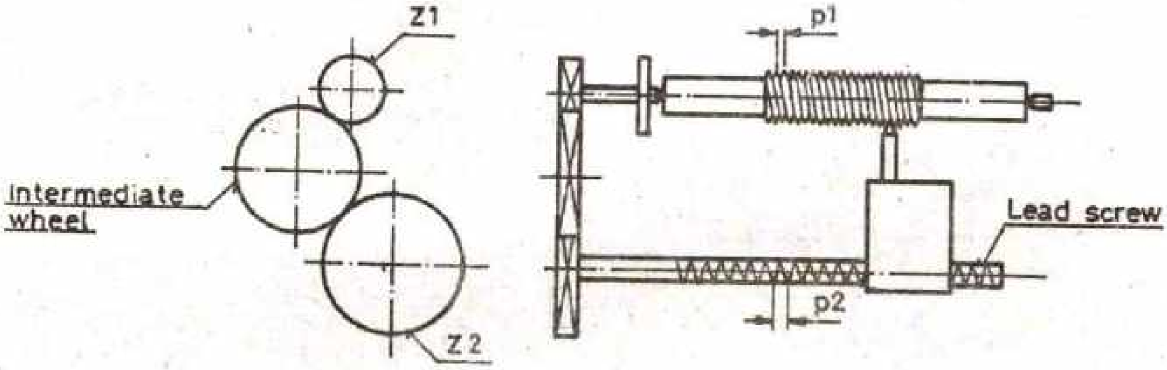
c) Calculate the total time if three pieces are to be produced.
Set-up time = 18 min/piece, auxiliary time = 3 min/piece, Delay time = 15 % of machining time.

CHAPTER 28

CHANGE GEAR CALCULATIONS

For cutting a thread on the lathe the feed of the thread cutting tool must coincide with the pitch of the thread. The required transmission ratio between the driving spindle and the driven lead screw can be obtained by using a wheel gear. The number of teeth of the required wheel gears can be calculated by the following method.

28.1 Single gearing



Transmission ratio = pitch ratio

$$\frac{\text{No. of teeth on driving gear}}{\text{No. of teeth on driven gear}} = \frac{\text{pitch of the thread to be cut}}{\text{pitch of lead screw}}$$

Formula 1:

$$\boxed{\frac{z_1}{z_2} = \frac{p_1}{p_2}}$$

Note: For remembering the formula, imagine the lathe bed as the fraction line !

Example: Lead screw pitch p_2 = 5 mm

pitch p_1 of thread to be cut = 2 mm

available gears have 20, 25, 30, 35 .... 120 teeth.

Find suitable gears for cutting the threads.

Solution: $\frac{z_1}{z_2} = \frac{p_1}{p_2} = \frac{2\ \text{mm}}{5\ \text{mm}} = \frac{2}{5}$

Gears must be chosen so that $\frac{z_1}{z_2} = \frac{2}{5}$

Now the numerator and denominator must be multiplied by the common number so that we get sizes of available gears, e.g.

$$\frac{z_1}{z_2} = \frac{2 \times 10}{5 \times 10} = \underline{\underline{\frac{20}{50}}}$$

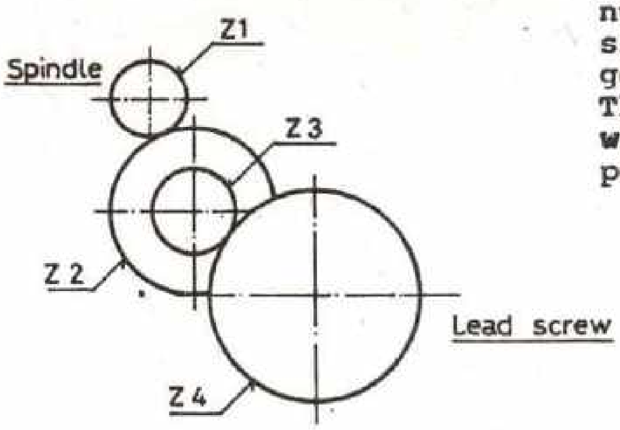
Result: Choose gears with 20 and 50 teeth.

Note: The number of teeth of the intermediate gear has no influence on the transmission ratio.
This gear only changes the direction of rotation.

28.2 Compound gearing

If the pitch ratio is greater than 1 : 6 we cannot find a suitable pair of wheel gears.
Therefore we have to make the ratio into two multiple factors. That means we have to separate the numerator and the denominator each into 2 factors.

These 4 factors are to be multiplied with a common number so that we get the size of 4 suitable wheel gears.
This system of arranging wheel gears is called compound gearing.



$$\frac{\text{Product of No. of teeth on driving gears}}{\text{Product of No. of teeth on driven gears}} = \frac{\text{Pitch of thread to be cut}}{\text{Pitch of lead screw}}$$

Formula 2: $$\boxed{\frac{z_1 \times z_3}{z_2 \times z_4} = \frac{p_1}{p_2}}$$

Example: Work pitch $p_1 = 1.5$ mm

Lead screw pitch $p_2 = 12$ mm

Gears vary by 5 teeth from 20 150

Find suitable gears to be used for thread cutting.

$$\frac{z_1}{z_2} = \frac{1.5 \text{ mm} \times 20}{12 \text{ mm} \times 20} = \frac{30}{240}$$

A gear with 30 teeth is available but none with 240. We must use a compound gear.

$$\frac{z_1 \times z_3}{z_2 \times z_4} = \frac{p_1}{p_2} = \frac{30}{240} = \frac{5 \times 6}{12 \times 20} = \underline{\underline{\frac{25 \times 30}{60 \times 100}}}$$

Result: Gears having 25, 60, 30 and 100 teeth shall be used.

Note: After calculation we have to check the result by the assembly condition.

28.3 To cut inch threads with an inch lead screw

If inch threads are required to be cut on a machine with an inch lead screw pitch, then both pitches are to be entered into the formula in inches:

Example: Threads to be cut on the workpiece have 8 threads per inch,
lead screw has 4 threads per inch,
gears available vary by 5 teeth from 20 onwards.

Find suitable gears.

Solution:

$$\frac{z_1}{z_2} = \frac{p_1}{p_2} = \frac{1" \times 4}{8 \times 1"} = \frac{4}{8} = \underline{\underline{\frac{20}{40}}}$$

Result: Gears shall have 20 and 40 teeth.

28.4 To cut inch threads with a metric lead screw

If inch threads are required to be cut on a machine with a metric lead screw, then both pitches are to be entered into the formula in millimeters.

Example: Threads to be cut on the workpiece have 5 threads per inch,
lead screw pitch p_2 = 12 mm,
gears available vary by 5 teeth from 20 ... 150 with an additional gear having 127 teeth.

Find suitable gears for cutting the threads.

Solution:

$$\frac{z_1}{z_2} = \frac{p_1}{p_2} = \frac{25.4 \text{ mm}}{5 \times 12 \text{ mm}} = \underline{\frac{127}{300}}$$

A gear with 127 teeth is available but none with 300. We must use a compound gear.

$$\frac{z_1 \times z_3}{z_2 \times z_4} = \frac{p_1}{p_2} = \frac{127}{300} = \frac{1 \times 127}{5 \times 60} = \underline{\frac{20 \times 127}{100 \times 60}}$$

Assembly condition:

$$(z_1 + z_2) - z_3 \geqq 15$$

$$(20 + 100) - 127 \geqq 15$$

$$120 - 127 \geqq 15$$

The conditions for assembling the gears are not fulfilled, therefore we have to choose the other sizes.

$$\frac{1 \times 127}{5 \times 60} = \frac{30 \times 127}{150 \times 60}$$

$$(30 + 150) - 127 \geqq 15$$

$$(127 + 60) - 150 \geqq 15$$

$$37 \geqq 15$$

Result: Choose gears with 30, 150, 127 and 60 teeth.

The sum of the number of teeth on the gears $z_1 + z_2$ must be greater than the number of teeth on the gear z_3 by at least 15 teeth.

Similarly the sum of the number of teeth on the gears $z_3 + z_4$ must be greater than the number of teeth on the gear z_2 by at least 15 teeth.

If it is less we would not be able to assemble the gears.

Example:

$$\begin{aligned}(z_1 + z_2) - z_3 &\geqq 15\\ (25 + 60) - 30 &\geqq 15\\ 85 - 30 &\geqq 15\\ \underline{55} &\underline{\geqq 15}\end{aligned}$$

$$\begin{aligned}(z_3 + z_4) - z_2 &\geqq 15\\ (30 + 100) - 60 &\geqq 15\\ 130 - 60 &\geqq 15\\ \underline{70} &\underline{\geqq 15}\end{aligned}$$

This compound can be assembled !

28.5 To cut metric threads with an inch lead screw

If metric threads are required to be cut on a machine with an inch lead screw pitch, then both pitches are to be entered into the formula in millimeters.

Example: Threads to be cut on the workpiece have 2 mm pitch, lead screw has 4 threads per inch, gears available vary by 5 teeth from 20 onwards with an additional gear wheel having 127 teeth.

Find suitable gears for cutting the thread.

Solution: the lead screw pitch in mm:

$$p_2 = \frac{25.4\text{ mm}}{4}$$

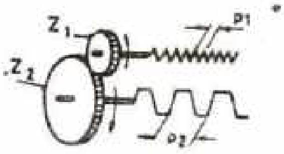
the transmission ratio:

$$\frac{z_1}{z_2} = \frac{p_1}{p_2} = \frac{2\text{ mm} \times 4}{25.4\text{ mm}} = \frac{8}{25.4} = \underline{\underline{\frac{40}{127}}}$$

Result: Choose gears with 40 and 127 teeth.

Exercises:

1) A screw with 5 mm pitch is to be cut. The lead screw pitch is 6 mm.
Choose suitable gears from the range 20, 24, 28 .... 160 teeth.



2) A screw with 1.5 mm pitch is to be cut on a lathe with lead screw pitch 5 mm. Find suitable gears from a range varying by 5 teeth from 20 .... 150 teeth.

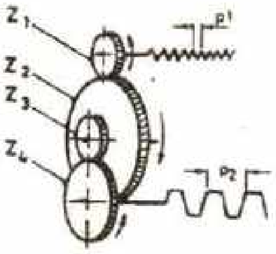
3) Threads are to be cut on a lathe with lead screw pitch 6 mm: M 8, M 12, M 20, M 42.
Find suitable gears from a range varying by 5 teeth from 20 .... 120.

4) The lead screw on lathe has 4 threads per inch. A screw with 5 mm pitch is to be cut. Gears with 25, 30, 35 .... 130 teeth are available as well as one with 127 teeth.
Which gears should be selected?

5) A screw with 11 threads per inch is to be cut on a lathe with a lead screw having 8 threads per inch. The available gears are the same as in 4).
Which shall be used?

6) The lead screw on a lathe has a pitch of 6 mm. A screw with 4 threads per inch is to be cut.
Choose suitable gears from a series with 20, 25, 30 .... 160 teeth as well as one with 127 teeth.

7) A screw with a pitch of 0.5 mm is to be cut on a lathe, the lead screw of which has a 6 mm pitch. The available gears are the same as in 6).
Which gears shall be used?



8) A lathe lead screw has a 5 mm pitch. The gears vary by 5 teeth from 20 .... 120, with an additional gear having 127 teeth.
A screw with 8 threads per inch is to be cut.
Find the suitable gear combination.

9) A 1 mm pitch screw is to be cut on a lathe with a lead screw having 4 threads per inch. The available gears vary by 4 teeth from 20 .... 132, and an additional gear having 127 teeth.
Choose the appropriate gears to be used.

IMPORTANT SI-UNITS

kg = unit of mass (1 kg = 1 000 g)
m = unit of length (1 m = 10 dm = 100 cm = 1 000 mm)
μm = unit of length (1 000 μm = 1 mm)
m^3 = unit of volume (1 m^3 = 1 000 dm^3)
l = unit of volume, fluid (1 l = 1 dm^3 = 1 000 cm^3)
m^2 = unit of area (1 m^2 = 10 000 cm^2; 1 cm^2 = 100 mm^2)
sec = unit of time (60 sec = 1 minute; 60 min = 1 hour)
N = unit of force (9.81 N = 1 kg_p)
J = unit of work, torque (1 Joule = 1 Newton metre = appr. 0.1 kgm)
W = unit of power (1 000 watts = 1 kilowatt)

IMPORTANT SYMBOLS

A = area, surface area, cross sectional area
l, L = length, total length
d, D = diametre, big diametre
h, H = height, thickness, total height
γ = specific gravity
w = weight

F = force
t = time
P = power
M = torque
p = pressure
σ = tensile strength
τ = shearing strength
W = work

d_o = pitch diametre
m = module
z = no. of teeth (of gear)
p = pitch, circular pitch
C = centre-to-centre distance
TT = rate of tool travel
i = ratio, transmission ratio, gear ratio

OS = tailstock offset
K = taper ratio
α = taper angle
$\alpha/2$ = setting angle

v = speed, velocity
CS = cutting speed
s = distance, feed
T = total time
n = no. of revolutions, no. of turns
t_m = machining time

ہُنر سکھاؤ

بے روزگاری مُکاؤ